Looking Ahead to Chapter 10

Focus

In Chapter 10, you will learn about polynomials, including how to add, subtract, multiply, and divide polynomials. You will also learn about polynomial and rational functions.

Chapter Warm-up

Answer these questions to help you review skills that you will need in Chapter 10.

Solve each two-step equation.

1. \[5x + 3 = 2x - 12\]
   \[x = -5\]

2. \[4x = 7 - 3x\]
   \[x = 1\]

3. \[\frac{x}{5} + 3 = \frac{x}{10}\]
   \[x = -30\]

Simplify each expression.

4. \[(3^2)^5\]
   \[3^{10}\]

5. \[x^7 \cdot x^{-5}(x^2)^{-3}\]
   \[\frac{1}{x^2}\]

6. \[\left(\frac{xy^3}{y^2}\right)^{-1}\]
   \[\frac{1}{xy^7}\]

Read the problem scenario below.

You have just planted a flower bed in a 3-foot wide by 5-foot long rectangular section of your yard. After planting some flowers, you decide that you would like to add to the width of your garden so that there is more space to plant flowers. Let \(x\) represent the amount added to the length of your garden.

7. Represent the area of your expanded garden with an expression in two ways by using the distributive property.
   \[5(3 + x) = 15 + 5x\]

8. What is the area of your garden if you expand the width by 4 feet?
   \[5(3 + 4) = 5(7) = 35\] square feet

Key Terms

- polynomial: p. 458
- term: p. 458
- coefficient: p. 458
- degree of a term: p. 458
- degree of the polynomial: p. 458
- monomial: p. 459
- binomial: p. 459
- trinomial: p. 459
- standard form: p. 459
- Vertical Line Test: p. 461
- combine like terms: p. 464
- distributive property: p. 464
- area model: p. 468

- divisor: p. 472
- dividend: p. 472
- remainder: p. 472
- FOIL pattern: p. 476
- square of a binomial sum: p. 477
- square of a binomial difference: p. 479
- factoring out a common factor: p. 483
- difference of two squares: p. 487
- perfect square trinomial: p. 487
- polynomial expression: p. 488
- greatest common factor: p. 488
- factoring by grouping: p. 489
- algebraic fraction: p. 491
- rational expression: p. 491
- excluded or restricted value: p. 491
- rational equation: p. 492
- extraneous solutions: p. 494
- Fundamental Theorem of Algebra: p. 497
- synthetic division: p. 500
Chapter 10

Polynomial Functions

Stained glass windows have been used as decoration in homes, religious buildings, and other buildings since the 7th century. In Lesson 10.4, you will find the area of a stained glass window.

10.1 Water Balloons
Polynomials and Polynomial Functions  p. 457

10.2 Play Ball!
Adding and Subtracting Polynomials  p. 463

10.3 Se Habla Español
Multiplying and Dividing Polynomials  p. 467

10.4 Making Stained Glass
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10.5 Suspension Bridges
Factoring Polynomials  p. 481

10.6 More Factoring
Factoring Polynomial Expressions  p. 487

10.7 Fractions Again
Rational Expressions  p. 491

10.8 The Long and Short of It
Polynomial Division  p. 497
Objectives
In this lesson, you will:
- Identify terms and coefficients of polynomials.
- Classify polynomials by the number of terms.
- Classify polynomials by degree.
- Write polynomials in standard form.
- Use the Vertical Line Test to determine whether equations are functions.

Key Terms
- polynomial
- term
- coefficient
- degree of a term
- degree of the polynomial
- monomial
- binomial
- trinomial
- standard form
- Vertical Line Test

Lesson Overview
Within the context of this lesson, students will be asked to:
- Identify if an expression is a polynomial.
- Identify terms and coefficients of polynomials.
- Classify polynomials by the number of terms as well as by the highest degree term.
- Write polynomials in standard form.
- Apply the Vertical Line Test to graphs of equations to determine whether the graphed equations are functions.

Essential Questions
The following key questions are addressed in this section:
1. What is a polynomial?
2. What is vertical motion?
3. How can you determine the coefficient of a term?
4. How can you determine the degree of a term? How can you determine the degree of a polynomial?
5. How can you determine whether a graphed equation is a function?
6. How can you write a function in standard form?
7. What is a monomial? What is a binomial? What is a trinomial?
Warm Up

Place the following questions or an applicable subset of these questions on the board before students enter class. Students should begin working as soon as they are seated.

The vertical motion model for a ball hit by a baseball bat is \( y = -16t^2 + 40t + 2.5 \), where \( y \) is the height in feet and \( t \) is the time in seconds. Answer each question. Recall that the general equation for a vertical motion model is \( y = -16t^2 + vt + h \), where \( v \) is the initial velocity in feet per second and \( h \) is the initial height in feet.

1. What is velocity? Velocity is the speed and direction in which an object is moving.
2. What is the initial velocity? 40 feet per second.
3. What does the constant 2.5 represent in the equation? The initial height from where the object was thrown was 2.5 feet off of the ground.
4. What is the height of the ball 1 second after the ball was hit? 26.5 feet
5. What shape will the graph of this equation look like? A parabola that opens down.
6. What is the vertex point of the parabola? The vertex point is at (1.25, 27.5).
7. What is the meaning of the vertex point for the parabola? This means that the ball is at the highest point of 27.5 feet, 1.25 seconds after the ball is hit.
8. What is the axis of symmetry for this parabola? The axis of symmetry of this parabola is \( x = 1.25 \).
9. When would the ball hit the ground? The ball will hit the ground about 2.56 seconds after it is hit. (Assuming no one else catches the ball before it hits the ground.)

Motivator

Begin the lesson with the motivator to get students thinking about the topic of the upcoming problem. This lesson is about the path of a tossed water balloon. The motivating questions are about tossing water balloons.

Ask the students the following questions to get them interested in the lesson.

- What is a water balloon?
- Why would someone fill a balloon with water?
- When have you tossed water balloons?
- What did the path of the balloon look like when you threw it?
Problem 1

Students will investigate vertical motion for a thrown water balloon.

Grouping

Ask for a student volunteer to read the Scenario and Problem 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Complete part (A) of Problem 1 together as a whole class. When the students understand the situation, have them work together in small groups to complete parts (B) through (F) of Problem 1. Then call the class back together to discuss and present their work for parts (B) through (F) of Problem 1.

Guiding Questions

■ What information is given in this problem?
■ What is the meaning of velocity?
■ What is vertical motion?
■ What formula did you find in Chapter 8 to model vertical motion?
■ In the formula \( y = -16t^2 + vt + h \), what is the meaning of the variable \( t \)?
■ What is the meaning of the variable \( y \)?
■ What is the meaning of the variable \( v \)?
■ What is the meaning of the variable \( h \)?
■ Will the water balloon travel in a horizontal line across the field? What will the path of the balloon look like?

SCENARIO

On a calm day, you and a friend are tossing water balloons in a field trying to hit a boulder in the field. The balloons travel in a path that is in the shape of a parabola.

Problem 1

Ready, Set, Launch

A. On your first throw, the balloon leaves your hand 3 feet above the ground at a velocity of 20 feet per second. Use what you learned in Chapter 8 about vertical motion models to write an equation that gives the height of the balloon in terms of time.

\[
y = -16t^2 + 20t + 3
\]

What is the height of the balloon after one second?

B. What is the height of the balloon after one second? Show your work and use a complete sentence to explain your answer.

\[
y = -16(1)^2 + 20(1) + 3 = -16 + 20 + 3 = 7
\]

The balloon is at a height of 7 feet on the ground after one second.

C. What is the height of the balloon after two seconds? Show your work and use a complete sentence in your answer.

\[
y = -16(2)^2 + 20(2) + 3 = -64 + 40 + 3 = -21
\]

The balloon is on the ground after two seconds.

D. On your second throw, the balloon leaves your hand at ground level at a velocity of 30 feet per second. Write an equation that gives the height of the balloon in terms of time.

\[
y = -16t^2 + 20t
\]

What is the height of the balloon after one second?

E. What is the height of the balloon after one second? Show your work and use a complete sentence in your answer.

\[
y = -16(1)^2 + 30 = -16 + 30 = 14
\]

The balloon is at a height of 14 feet on the ground after one second.

F. What is the height of the balloon after two seconds? Show your work and use a complete sentence in your answer.

\[
y = -16(2)^2 + 30(2) = -64 + 60 = -4
\]

The balloon is on the ground after two seconds.

---

Take Note

The vertical motion model is \( y = -16t^2 + vt + h \), where \( t \) is the time in seconds that the object has been moving, \( v \) is the initial velocity (speed) in feet per second of the object, \( h \) is the initial height in feet of the object, and \( y \) is the height in feet of the object at time \( t \).
Investigate Problem 1

Students will compare two polynomials used to model vertical motion.

Take Note

Whole numbers are the numbers 0, 1, 2, 3, and so on.

Grouping

Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Have students work together in small groups to complete Question 1. Then call the class back together to have the students discuss and explain their work for Question 1.

Just the Math

Students will be formally introduced to terminology for polynomials in Questions 2 and 3. It is important for students to recognize that terms in a polynomial are separated by addition. This is true because a polynomial such as \(x^2 - 2\) can be written as \(x^2 + (-2)\). So, \(x^2\) and \(-2\) are each terms of this polynomial.

Grouping

Ask for a student volunteer to read Question 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Complete Questions 2 and 3 together as a whole class. Then have students work together in small groups to complete Questions 4 and 5.

Guiding Questions

- What is a polynomial? What are the important characteristics of a polynomial?
- What is a term? What is a coefficient?
- In the polynomial \(-3x^2 + 4x + (-5)\) list the terms? What is the coefficient of \(x^2\)? What is the coefficient of \(x\)?
- What is a degree of a term? What is a degree of a polynomial?
- How can you find the degree of a term? How can you find the degree of a polynomial?
Explore Together

Investigate Problem 1

Students will classify polynomials according to the number of terms in each polynomial.

Call the class back together to have the students discuss and present their work for Question 4.

Grouping

Ask for a student volunteer to read Question 5 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Questions 5 and 6.

Guiding Questions

■ What is a term for a polynomial?
■ How can you find the number of terms?
■ What is a monomial? What is a binomial? What is a trinomial?
■ Why do you think that we only have special names for polynomials with 1, 2, or 3 terms?

Call the class back together to have the students discuss and present their work for Questions 5 and 6.

Just the Math

The concept of the standard form of a polynomial will be introduced in Question 7.

Grouping

Ask for a student volunteer to read Question 7 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Questions 7 and 8.

Guiding Questions

■ What does it mean for something to have a standard or to be standardized?
■ Why is it important for us to be able to write polynomials in a standard form? What would happen if several people were considering a polynomial to represent the motion of the space shuttle, but each had the polynomial represented in a different way?
■ How can you write a polynomial in standard form?

Notes

Only the basic approach to standard form is presented in this problem. Students will not yet be asked to write a polynomial that has more than one variable in standard form.

Investigate Problem 1

What kind of expression is a polynomial of degree 2? Give an example and use a complete sentence to explain your reasoning.

Sample Answer: $x^2$; A polynomial of degree 2 is a quadratic expression.

A polynomial of degree 3 is a called a cubic polynomial. Write an example of a cubic polynomial.

Sample Answer: $x^3$

5. For each of your polynomial models in parts (A) and (D), find the number of terms in the model. Use a complete sentence in your answer.

The polynomial model for the first throw has three terms and the model for the second throw has two terms.

Polynomials with only one term are monomials. Polynomials with exactly two terms are binomials. Polynomials with exactly three terms are trinomials. Classify each polynomial model in parts (A) and (D) by its number of terms. Use a complete sentence in your answer.

The polynomial model for the first throw is a trinomial and the polynomial model for the second throw is a binomial.

6. Give an example of a monomial of degree 3.

Sample Answer: $4x^3$

Give an example of a trinomial of degree 5.

Sample Answer: $x^5 + 2x - 3$

7. Just the Math: Standard Form of a Polynomial

Later in this chapter, we will be adding, subtracting, multiplying, and dividing polynomials. To make this process easier, it is helpful to write polynomials in standard form. A polynomial is written in standard form by writing the terms in descending order, starting with the term with the greatest degree and ending with the term with the least degree. Write each polynomial in standard form.

\[ 6 + 5x \]
\[ 5x + 6 \]
\[ 4 + 3x + 4x^2 \]
\[ 4x^2 + 3x + 4 \]
\[ 5 - 6x^4 \]
\[ -6x^4 + 5 \]
\[ 7 - x^2 \]
\[ -x^2 + 7 \]
\[ 4 + 3x^2 + 9x - x^2 \]
\[ -x^3 + 3x^2 + 9x + 4 \]
\[ x^4 - 4x^2 + 16 \]
\[ x^6 - 4x^3 + 16 \]
Investigate Problem 1

Students will complete a summary question to review the words polynomials, terms, and degrees.

Call the class back together to have the students discuss and present their work for Questions 7 and 8.

Key Formative Assessments
- What are the requirements for an expression to be considered a polynomial?
- How can you determine whether an expression is a polynomial?
- How can you determine where a term begins and where the term ends?
- How can you determine the number of terms in a polynomial?
- What are the requirements for a polynomial to be in standard form?

Problem 2

Grouping

Ask for a student volunteer to read Problem 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) through (C) of Problem 2.

Guiding Questions
- What information is given in this problem?
- How is this problem similar to Problem 1?
- How is this problem different from Problem 1?
- How can you write an equation to model the motion for the balloon thrown by your friend?

If the students finish Problem 1 during the class period and do not have enough time to complete Problem 2 during class, parts (A) through (C) of Problem 2 can be assigned as part of a homework assignment. At the beginning of the next class session, call the students together to discuss and present parts (A) through (C) of Problem 2 and then continue with the problem as suggested on the next few pages.
Problem 2

Students will graph a parabolic function representing the height of a water balloon as a function of the time after the balloon was thrown.

Call the class back together to have the students discuss and present their work for parts (A) through (C) of Problem 2.

**Key Formative Assessments**
- What two quantities were compared for your graph in part (C)?
- What is the unit for time? What is the unit for height?
- What is the shape of the graph?
- Does this shape make sense for this problem situation? Why or why not?
- Why does the parabola open downward rather than upward?
- How many y-intercepts are there?
- What is the meaning of the y-intercept for your graph?
- How many x-intercepts are there?
- Why is there only one x-intercept?
- What is the meaning of the x-intercept?

**Investigate Problem 2**

**Grouping**
- Ask for a student volunteer to read Questions 1 and 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Complete Questions 1 and 2 together as a whole class.

**Guiding Questions**
- What is a function? What does it mean for every input value to have exactly one output value?
- What is the Vertical Line Test and how can you use it? Does the vertical line have to pass through just one point at a selected x-value, or does it have to pass through exactly one point at each x-value?

**Problem 2** The Balloon’s Path

C. Create a graph of the model to see the path of the balloon on the grid below. First, choose your bounds and intervals. Be sure to label your graph clearly.

<table>
<thead>
<tr>
<th>Variable quantity</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time</td>
<td>0</td>
<td>3.75</td>
<td>0.25</td>
</tr>
<tr>
<td>Height</td>
<td>0</td>
<td>30</td>
<td>2</td>
</tr>
</tbody>
</table>

Sample Answer: Yes. For every input, there is only one output.

1. Is the equation that you wrote in part (A) a function? How do you know? Use a complete sentence in your answer.

Sample Answer: Yes. For every input, there is only one output.

2. Just the Math: Vertical Line Test You can use a graph to determine whether an equation is a function. The Vertical Line Test states that an equation is a function if you can pass a vertical line through any part of the graph of the equation and the line intersects the graph, at most, one time. Consider your graph in part (C). Does your graph pass the Vertical Line Test?

Yes.
Investigate Problem 2

Students will determine whether a given graph is the graph of a function by applying the Vertical Line Test.

**Grouping**

Ask for a student volunteer to read Question 3 aloud. Pose the Guiding Question below to verify student understanding. Have students work together in small groups to complete Question 3.

**Guiding Question**

■ How can you apply the Vertical Line Test to determine whether a graph is a graph of a function?

**Common Student Errors**

Some students don’t naturally associate the Vertical Line Test with the requirement for a function to have exactly one $y$-value for each individual $x$-value in the domain. They can see it as an easy trick without understanding the concept. Be sure to ask enough questions to verify their understanding of functions and their graphs.

Call the class back together to have the students discuss and present their work for Question 3.

**Key Formative Assessments**

■ What is a monomial? What is a binomial? What is a trinomial?
■ Are coefficients always whole numbers? Are they always positive?
■ How can you determine the degree of a polynomial? Is it possible to have a degree of polynomial that is $\frac{1}{2}$?
■ What is a coefficient in a polynomial?
■ How can you write a polynomial in standard form?
■ How can you determine whether a graph is the graph of a function?
■ Are all linear graphs functions? What type of line is not a function? Is a horizontal line a function?
Wrap Up

Close

- Review all key terms and their definitions. Include the terms polynomial, term, coefficient, degree, monomial, binomial, trinomial, standard form, and Vertical Line Test.

- You may also want to review any other vocabulary terms that were discussed during the lesson which may include velocity, speed, vertical motion, x-value, y-value, x-intercept, y-intercept, whole number, exponent, function, linear function, quadratic function, cubic function, domain, range, equation, and expression.

- Remind the students to write the key terms and their definitions in the notes section of their notebooks. You may also want the students to include examples.

- Ask the students to each suggest a monomial, a binomial, a trinomial, and an expression or equation that is not a polynomial by writing them down on paper. While they are doing that, divide the front board into 4 sections. Label the first with the name monomial, the second with binomial, the third with trinomial, and the fourth with not a polynomial. Have the students then write their suggestions in the appropriate place on the front board. Begin a discussion by asking several students to choose one of the equations that is not a polynomial and explain what polynomial requirement it fails. Next, ask the students to verify the classifications of monomial, binomial, and trinomial. If they disagree with any of the suggestions, have them explain why. Then, ask the students to identify the polynomials that have a degree of 1, then repeat for a degree of 2, etc. Ask which polynomials are written in standard form. If any are not in standard form, ask volunteers to rewrite them in standard form. Finally, ask them to determine which polynomials are functions and which are not functions. Have the students explain their answers.

Follow Up

Assignment

Use the Assignment for Lesson 10.1 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Open-Ended Writing Task

Have the students each write 2 monomials, 2 binomials, and 2 trinomials. Have the students then write each of their polynomials in standard form, determine whether each is a function, identify the coefficients of each term, identify the degree of each term, and identify the degree of each polynomial. At the beginning of the next class session, have the students trade their work and verify the answers for their partners.
Reflections
Insert your reflections on the lesson as it played out in class today.

What went well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

What did not go as well as you would have liked?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

How would you like to change the lesson in order to improve the things that did not go well and capitalize on the things that did go well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
10.2 Play Ball!
Adding and Subtracting Polynomials

Learning By Doing Lesson Map

Get Ready

Objectives
In this lesson, you will:
■ Add polynomials.
■ Subtract polynomials.

Key Terms
■ combine like terms
■ distributive property
■ add
■ subtract

Sunshine State Standards
MA.912.A.2.13
Solve real-world problems involving relations and functions.

MA.912.A.4.2
Add, subtract, and multiply polynomials.

MA.912.A.10.1
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

MA.912.A.10.2
Decide whether a solution is reasonable in the context of the original situation.

Lesson Overview
Within the context of this lesson, students will be asked to:
■ Evaluate two polynomials for the same x-value, then find the sum of the results.
■ Add two polynomials, then evaluate the sum for a given value of x.
■ Compare the results for the two sums.
■ Add polynomials.
■ Subtract polynomials.
■ Simplify expressions requiring addition and subtraction of polynomials.

Essential Questions
The following key questions are addressed in this section:
1. What is the distributive property?
2. What are like terms?
3. How can you combine like terms?
4. How can you add polynomials?
5. How can you subtract polynomials?
Warm Up

Place the following questions or an applicable subset of these questions on the board before students enter class. Students should begin working as soon as they are seated.

Simplify each expression.

1. \(10x - 4x\)   \(6x\)  
2. \(15x - 2x + 3x\)   \(16x\)  
3. \(\frac{30 + 6}{2 \cdot 6}\)   \(3\)  
4. \(\frac{5(10 + 4) - 16}{6}\)   \(9\)  
5. \(3(2 - x)\)   \(6 - 3x\)  
6. \(3(x^2 + x)\)   \(3x^2 + 3x\)  
7. \(-\frac{8}{6} \left(\frac{12}{16}\right)\)   \(-1\)  
8. \(-5(7x^3 + 2x^2 + 3x)\)  
9. \(3(x + 2) - 4\)   \(3x^2 + 2\)  
\(-35x^3 - 10x^2 - 15x\)

Motivator

Begin the lesson with the motivator to get students thinking about the topic of the upcoming problem. This lesson is about attendance at baseball games. The motivating questions are about attending baseball games.

Ask the students the following questions to get them interested in the lesson.

- Have you attended a major league baseball game?
- What stadium(s) have you attended?
- What is your favorite stadium? Why?
- What is the closest major league baseball stadium and team to our school?
- Is that team part of the National League or the American League?
Problem 1

Students will evaluate cubic functions for given values of $x$. Students will then find the sum of the evaluated functions for the same $x$-value.

Grouping

Ask for a student volunteer to read the Scenario and Problem 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) and (B) of Problem 1. Then call the class back together to have the students discuss and present their work for parts (A) and (B).

Guiding Questions

- What information is given in this problem?
- What is the degree of each given polynomial representing attendance?
- What does $x$ represent in each equation? What does $y$ represent in each equation?
- How are the equations similar? How are the equations different?
- What are you asked to do in this problem?
- How can you determine the $x$-value for a given year?
- How can you evaluate a given function for a specified value of $x$?

Common Student Errors

Students will often multiply the coefficient of a variable term by the value for the variable before evaluating the exponent. Be ready to remind the students of the order of operations for evaluating an expression.

Students also often struggle with numbers of large magnitude. It is difficult for them to comprehend these numbers because they can’t visualize them easily. Remind the students to check their answers for reasonableness.

**Scenario**

A friend of yours loves baseball and plans to visit every major league baseball park in the country. The major league is made up of two divisions, the National League and the American League.

**Problem 1 Batter Up**

Your friend recently read an article that stated that the attendance at National League baseball games from 1990 through 2001 can be modeled by the function

$$y = -86,584x^3 + 1,592,363x^2 - 5,692,368x + 24,488,926$$

where $x$ is the number of years since 1990 and $y$ is the number of people who attended. The article also stated that the attendance at American League baseball games from 1990 through 2001 can be modeled by the function

$$y = -56,554x^3 + 1,075,426x^2 - 4,806,571x + 30,280,751$$

where $x$ is the number of years since 1990 and $y$ is the number of people who attended.

A. Find the attendance for each league in 1995. Show your work and use a complete sentence in your answer.

- **National League:**
  $$y = -86,584(5)^3 + 1,592,363(5)^2 - 5,692,368(5) + 24,488,926$$
  $$y = 25,013,161$$

- **American League:**
  $$y = -56,554(5)^3 + 1,075,426(5)^2 - 4,806,571(5) + 30,280,751$$
  $$y = 26,064,296$$

In 1995, the attendance for the National League was 25,013,161 people and the attendance for the American League was 26,064,296 people.

B. Find the total attendance at all games in major league baseball in 1995. Show your work and use a complete sentence in your answer.

$$25,013,161 + 26,064,296 = 51,077,457$$

The total attendance in 1995 was 51,077,457 people.

How did you find your answer to part (B)? Use a complete sentence in your answer.

**Sample Answer:** I added the 1995 National League attendance to the 1995 American League attendance.
Problem 1

Investigate Problem 1

Students will add two polynomials by combining the like terms.

**Guiding Questions**
- How do parts (C) and (D) differ from parts (A) and (B)?
- How will you find the x-value for the year 2000?
- Call the class back together to have the students discuss and present their work for parts (C) and (D) of Problem 1.

**Key Formative Assessments**
- How could you find the predicted attendance for this year for each of the two major leagues in baseball?
- How many years since 1990 is it now? What value will we use for x?
- What is the predicted attendance for the National League this year? What is the predicted attendance for the American League this year? What is the total predicted attendance for this year in major league baseball?
- Do you think there is a simpler way to get a predicted total attendance than finding the individual values separately and then adding them together?

**Take Note**

Remember that you use a distributive property to combine like terms:

\[ 4x + 10x = x(4 + 10) \]
\[ = x(14) \]
\[ = 14x \]
Investigate Problem 1

Students will compare the result of finding the sum of two polynomials evaluated at the same \( x \)-value to the result of finding the sum of the polynomials and then evaluating at the same \( x \)-value.

Call the class back together to have the students discuss and present their work for Questions 1 through 3.

Key Formative Assessments

- How can you add two polynomials?
- What are like terms?
- Does the degree of each term have to be equal for the terms to be considered like terms?
- How can you combine like terms?
- How do you use the distributive property to combine like terms?
- Which method is easier, evaluating the individual attendance equations for a given year and then adding their values or adding the like terms first and then evaluating the new equation?
- What other situation can you think of in which it might be easier to add polynomials before evaluating?

Grouping

Ask for a student volunteer to read Question 4 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have the students complete Question 4 individually. Then have the students discuss and explain their work for Question 4 with the class.

Guiding Questions

- What are we asked to do in Question 4?
- How can you subtract polynomials?

Investigate Problem 1

3. Use your model to find the total attendance in 1995 and 2000. Show your work and use a complete sentence in your answer.

1995:
\[ y = -143,138(5)^3 + 2,667,789(5)^2 - 10,498,939(5) + 54,769,677 \]
\[ y = 51,077,457 \]

2000:
\[ y = -143,138(10)^3 + 2,667,789(10)^2 - 10,498,939(10) + 54,769,677 \]
\[ y = 73,421,187 \]

In 1995, the total attendance was 51,077,457 people and in 2000, the total attendance was 73,421,187 people.

How do these answers compare to those in Problem 1 parts (B) and (D)? Use a complete sentence in your answer.

Sample Answer: The answers are the same as those in Problem 1 parts (B) and (D).

4. You can also write a function that shows how many more people attended National League games each year than attended American League games. Describe how you can find this function. Use a complete sentence in your answer.

Sample Answer: You can find this function by subtracting the function for American League game attendance from the function for National League game attendance.

Complete the statement below that gives the function described above.

\[ y = (-86,584x^3 + 1,592,363x^2 - 5,692,368x + 24,488,926) - (-56,554x^3 + 1,075,426x^2 - 4,806,571x + 30,280,751) \]

Because you are subtracting one function from another function, you must subtract each term of the second function from the first function. To do this, use the distributive property to distribute the negative sign to each term in the second function.

\[ y = -86,584x^3 + 1,592,363x^2 - 5,692,368x + 24,488,926 - 56,554x^3 - 1,075,426x^2 + 4,806,571x - 30,280,751 \]

Common Student Errors

Some students may have difficulty subtracting polynomials. Remind the students that subtraction is the same as adding the opposite of the second quantity.
Investigate Problem 1

Students will practice adding and subtracting polynomials.

Grouping
Ask for a student volunteer to read Question 5 aloud. Have a student restate the problem.

If time permits during the class period, have students work together in small groups to complete Questions 5 and 6.

If time is limited at the end of the class session, you can assign Questions 5 and 6 as part of a homework assignment.

Common Student Errors
Students will frequently confuse unlike terms to be like terms because they have the same base. You may need to stop and briefly review terminology of the base and the exponent for powers. Remind the students that terms are only considered to be like if both the base and exponent match exactly.

Conversely, other students will incorrectly assume that terms are like only if the coefficient, the base, and the exponent all match exactly. It may help these students to be reminded that the coefficient is a counter. It tells us the number of that variable term that exists.

Call the class back together to have the students discuss and present their work for Questions 5 and 6.

Key Formative Assessments

- What are like terms?
- How can you use the distributive property to combine like terms?
- How can you add polynomials?
  - When is it useful to add polynomials?
- How can you subtract polynomials?
  - When is it useful to subtract polynomials?

5. What was the difference in attendance between the leagues in 1998? Which league had more people attending games in 1998? How do you know? Show your work and use complete sentences in your answer.

\[ y = -30,030(8)^2 + 516,937(8) - 885,797(8) - 5,791,825 \]

Sample Answer: The difference was 4,830,407 people.

6. Simplify each expression by finding the sum or difference. Show your work.

\[
\begin{align*}
(4x^2 + 3x - 5) + (x^2 - 8x + 4) &= (4x^2 + x^2) + (3x - 8x) + (-5 + 4) \\
&= 5x^2 - 5x - 1 \\
(2x^3 + 3x - 4) - (2x^2 + 5x - 6) &= (2x^3 - 2x^2) + (3x - 5x) + (-4 + 6) \\
&= -2x + 2 \\
(4x^3 + 5x - 2) + (7x^2 - 8x + 9) &= 4x^3 + 7x^2 + (5x - 8x) + (-2 + 9) \\
&= 4x^3 + 7x^2 - 3x + 7 \\
(9x^4 - 5) - (8x^4 - 2x^3 + x) &= (9x^4 - 8x^4) + 2x^3 - x - 5 \\
&= x^4 + 2x^3 - x - 5
\end{align*}
\]
Wrap Up

Close

- Review all key terms and their definitions. Include the terms combine like terms, distributive property, add, and subtract.

- You may also want to review any other vocabulary terms that were discussed during the lesson which may include order of operations, coefficient, terms, like terms, polynomial, function, evaluate, magnitude, sum, difference, base, exponent, and powers.

- Remind the students to write the key terms and their definitions in the notes section of their notebooks. You may also want the students to include examples.

- Ask the students to compare and contrast the process of adding polynomials and subtracting polynomials.

- Ask the students to write a polynomial with at least 4 terms. Ask several students to write their polynomial on the front board. Choose a pair of the polynomials and have the class find the sum and then the difference of those polynomials. Repeat this process with other pairs of polynomials on the board as many times as you think is needed to help the students fully understand adding and subtracting polynomials.

Follow Up

Assignment

Use the Assignment for Lesson 10.2 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Open-Ended Writing Task

Have the students create two polynomials with at least 3 terms each. Have the students write the polynomials in standard form and then find the sum and the difference of the two polynomials. Ask the students to write a few sentences to explain their opinion as to what effect the order of the two polynomials has for both the sum and for the difference. During the next class session, discuss their work and responses for this activity as a whole class.
Reflections
Insert your reflections on the lesson as it played out in class today.

What went well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

What did not go as well as you would have liked?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

How would you like to change the lesson in order to improve the things that did not go well and capitalize on the things that did go well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
**Objectives**

In this lesson, you will:
- Use an area model to multiply polynomials.
- Use distributive properties to multiply polynomials.
- Use long division to divide polynomials.

**Key Terms**
- area model
- distributive property
- divisor
- dividend
- remainder

**Sunshine State Standards**

**MA.912.A.2.13**
Solve real-world problems involving relations and functions.

**MA.912.A.4.2**
Add, subtract, and multiply polynomials.

**MA.912.A.10.1**
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

**MA.912.A.10.2**
Decide whether a solution is reasonable in the context of the original situation.

**Lesson Overview**

Within the context of this lesson, students will be asked to:
- Evaluate polynomials for a given value of $x$ and then multiply the results.
- Multiply the polynomials, then evaluate the product for the same given value of $x$.
- Compare the answer from each process and identify them as resulting in the same value.
- Follow a similar procedure for division of polynomials.
- Use long division to divide polynomials.
- Students will identify the divisor, dividend, and remainder when dividing polynomials.

**Essential Questions**

The following key questions are addressed in this section:

1. What is an area model?
2. What is the distributive property and how can it help you to multiply polynomials?
3. How can you multiply polynomials?
4. What is long division?
5. How can you use long division to divide polynomials?
**Show The Way**

**Warm Up**
Place the following questions or an applicable subset of these questions on the board before students enter class. Students should begin working as soon as they are seated.

**Use the properties of exponents to simplify each expression. Write your answer as a power.**

1. \(2^3 \cdot 2^4\)  
2. \(7^2 \cdot 7^5 \cdot 7\)

3. \(4^5 \cdot 4^5 \cdot 8^2 \cdot 8^3\)  
4. \(\frac{5^9}{5^7}\)

5. \(\frac{2}{2^3}\)  
6. \(\frac{(-8)^7}{(-8)^6}\)

**Write each percent as a decimal.**

7. 8% 0.08  
8. 41% 0.41

**Write each decimal as a percent.**

9. 0.16 16%  
10. 0.0007 0.07%

**Motivator**
Begin the lesson with the motivator to get students thinking about the topic of the upcoming problem. This lesson is about the number of students who take foreign language classes. The motivating questions are about foreign language classes.

Ask the students the following questions to get them interested in the lesson.

- What foreign language classes are offered at our school?
- Have you ever taken a foreign language class?
- What language did you study or are you studying?
- How did you choose which language to study?
- What other foreign language would you like our school to offer?
**Explore Together**

**Problem 1**

Students will evaluate two separate polynomials for a given x-value and then multiply the results.

**Grouping**

Ask for a student volunteer to read the Scenario and Problem 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) through (D) of Problem 1.

**Guiding Questions**

- What information is given in this problem?
- What is a foreign language?
- What does the equation \( y = -0.0005t^2 + 0.02t + 0.23 \) represent?
- What is the meaning of \( t \) in that equation?
- How would you interpret a \( y \)-value of 0.16?
- What \( x \)-value would represent the year 1982? How can you find the \( x \)-value for a given year?
- What does the equation \( y = -3t^3 + 134t^2 - 1786t + 18,398 \) represent?
- How would you interpret a \( y \)-value of 127? How would you interpret a \( y \)-value of 1356?
- What number is equivalent to 1356 thousand?

**Take Note**

In Lesson 3.3, you learned how to use the percent equation. The percent equation is an equation of the form \( a = pb \) where \( p \) is the percent in decimal form and the numbers that are being compared are \( a \) and \( b \).

**SCENARIO**

Your high school is trying to decide whether to offer more foreign language classes. The guidance counselor finds a model for the percent of high school students that were in a foreign language class in high school during the years from 1985 to 2000, which is:

\[ y = -0.0005t^2 + 0.02t + 0.23 \]

where \( t \) represents the number of years since 1980 and \( y \) is the percent (in decimal form) of all high school students who were in a foreign language class. The guidance counselor also finds that the total number of students in high school during the years from 1985 to 2000 can be modeled by:

\[ y = -3t^3 + 134t^2 - 1786t + 18,398 \]

where \( t \) represents the number of years since 1980 and \( y \) is the number of students in thousands.

**Problem 1**

**Learning a Foreign Language**

A. What percent of high school students were in a foreign language class in 1990? Show your work and use a complete sentence in your answer.

\[ y = -0.0005(10)^2 + 0.02(10) + 0.23 \]

\[ y = 0.38 \]

In 1990, 38% of high school students were in a foreign language class.

B. Find the total number of students in high school in 1990. Show your work and use a complete sentence in your answer.

\[ y = -3(10)^3 + 134(10)^2 - 1786(10) + 18,398 \]

\[ y = 10,938 \]

In 1990, 10,938 thousand students were in high school.

C. In 1990, how many students were in a foreign language class? Show your work and use a complete sentence in your answer.

Sample Answer: \( 0.38(10,938) = 4156.44 \)

In 1990, approximately 4156 thousand students were in a foreign language class.

D. Use complete sentences to explain how you found your answer to part (C).

Sample Answer: To find the number of students in a foreign language class, multiply the total number of students in high school by the percent that are in a foreign language class.

**Notes**

Students may also correctly answer part (B) with 10,938,000 students.

Students can also correctly complete part (C) using a proportion.

Call the class back together to have the students discuss and present their work for parts (A) through (D).
Explore Together

Investigate Problem 1

Students will use an area model to multiply two polynomials together.

Grouping
Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Complete Question 1 together as a class.

Just the Math
Question 2 will help visual learners to better comprehend multiplication of polynomials. You will need to explain to the students that the variable $x$ represents an unknown value. We can represent $x$ in the model by choosing any length. However, we must represent each length of $x$ with the same length. The constant of 1 represents the length of 1 unit. For instance, we might want it to represent 1 inch or 1 centimeter. Question 3 will extend their understanding to multiply polynomials symbolically.

Guiding Questions
- What is a geometric model?
- What is area?
- How can you find the area of a rectangle?
- What does $x$ represent in this situation?
- If $x$ is a positive number, which will be larger, $3x$ or $4x + 1$? Why?
- How can you use an area model to determine the product of two polynomials?
- How can you multiply polynomials symbolically?

Take Note
Recall the distributive properties of multiplication from Lesson 4.3:

- $a(b + c) = ab + ac$
- $a(b - c) = ab - ac$

Two other related distributive properties are:

- $(b + c)a = ba + ca$
- $(b - c)a = ba - ca$
When can you simplify the results from

\((5x^2 + 2x - 3)(x + 1)\)?

In Question 3, students could have started by multiplying each term of the polynomial \((x^2 - 3x + 2)\) by the binomial \((x + 1)\), and then simplifying. This will give the same product and may be easier for some students than the method suggested in Question 3.

**Grouping**

Ask for a student volunteer to read Question 4 aloud. Have a student restate the problem. Have the students complete Question 4 individually. Then discuss the answers together as a whole class.

**Guiding Questions**

- How can you multiply polynomials?
- What types of errors do you think are common in these questions?
- After you multiply each term of one polynomial by each term of the other polynomial, what should you do next?
- When can you simplify the results from multiplying polynomials?

**Common Student Errors**

Multiplying polynomials is a difficult concept for students. Because it is abstract and out of context, sign errors, multiplication errors, and general arithmetic errors are common. Remind the students to check over their work carefully. It is often helpful for the students to rewrite the original polynomials in terms of addition only rather than subtraction. For instance, to multiply \((x^2 - 5)(-x^2 - 4x - 2)\), students could rewrite the problem as \((x^2 + (-5))((-x^2) + (-4x) + (-2))\), then multiply. This might help to prevent sign errors.

To simplify each product, remember that multiplication is commutative (you can multiply numbers in any order) and use the properties of exponents that you learned in Chapter 9. For instance, \((2x)(5x) = 10x^2\). Complete the product of \(3x + 4x + 1\) below.

\[(3x)(4x + 1) = (3x)(4x) + (3x)(1) = \_12x^2 + \_3x\]

To multiply the polynomials \(x + 1\) and \(x^2 - 3x + 2\), you need to use the distributive property twice. First, use a distributive property to multiply each term of \(x + 1\) by the polynomial \(x^2 - 3x + 2\). Complete the first step below.

\[(x + 1)(x^2 - 3x + 2) = (x)(x^2 - 3x + 2) + (1)(x^2 - 3x + 2)\]

Now, distribute \(x\) to each term of \(x^2 - 3x + 2\) and distribute 1 to each term of \(x^2 - 3x + 2\). Complete the second step below:

\[(x + 1)(x^2 - 3x + 2) = x(x^2) + (-3x) + (x)(2) + (1)(x^2)\]

Now multiply and collect like terms. Show your work and write your answer as a polynomial in standard form.

\[(x + 1)(x^2 - 3x + 2) = x^3 - 3x^2 + 2x + x^2 - 3x + 2\]

\[= x^3 - 2x^2 - x + 2\]

4. What do you notice about the products of the terms in Question 3 when you used a distributive property the second time? Use a complete sentence in your answer.

Sample Answer: The products are formed by multiplying each term in one polynomial by each term in the other polynomial.

5. Find each product. Show all your work.

\[
\begin{align*}
2x(x + 3) &= 5x^2(7x - 1) \\
2x(x + 2)(3) &= 2x^2 + 6x \\
(5x^2(7x) - (5x^2)(1)) &= 35x^3 - 5x^2 \\
(x + 1)(x + 3) &= (x^2 - 4)(2x - 3) \\
(x + 1)(x + 1)(3) &= (x^3 - 4)(2x + (x^2 - 4)(3) \\
= x(x) + (1)(x) + (3) &= x^3(2x) + (4)(x) + x^3(3) + (4)(3) \\
&= x^2 + x + 3x + 3 &= 2x^2 + 8x + 3x - 12 \\
&= x^2 + 4x + 3 &= 2x^2 + 3x^2 - 8x - 12 \\
(x - 5)(x^2 + 3x + 1) &= (x - 5)(x^2) + (x - 5)(3x) + (x - 5)(1) \\
&= x^3 - 5(x^3) + x^3 - 5(3x) + x + 1 - 5(1) \\
&= x^3 - 5x^2 + 3x^2 - 15x + x - 5 \\
&= x^3 - 2x^2 - 14x - 5
\end{align*}
\]
Explore Together

Investigate Problem 1

Students will multiply the polynomials from the beginning of the lesson to determine the total number of students taking a foreign language class for any given year in the domain.

Grouping

Students will be working together in small groups to complete Questions 6 and 7.

Common Student Errors

Some students may not understand why they are multiplying the two polynomials in Question 6 to find the number of students who are taking a foreign language. Pose the Guiding Questions below to verify student understanding of this concept.

Guiding Questions

■ What is the function that represents the percent of the total number of high school students enrolled in a foreign language?
■ What is the function that represents the total number of high school students for any given year in the domain?
■ If 20% of the students were enrolled in a foreign language class and there were 1000 total students, how would you find the number of students enrolled in a foreign language class?
■ Why would you multiply those values?

Call the class back together to have the students discuss and present their work for Questions 5 through 7.

Notes

Students may give the answer to Question 7 as 5,280,000. Be sure to have the students read this number correctly as five million two hundred eighty thousand students.

Investigate Problem 1

6. Use multiplication to find the function that gives the number of students in thousands that were in a foreign language class during the years from 1985 to 2000.

\[ (0.0005t^2 + 0.02t + 0.23)(-3t^3 + 134t^2 - 1786t + 18,398) \]

First, distribute each term of the polynomial for the percent to each term of the polynomial for the total number of students.

\[ -0.0005t^2(-3t^3 + 134t^2 - 1786t + 18,398) + 0.02t(-3t^3 + 134t^2 - 1786t + 18,398) + 0.23(-3t^3 + 134t^2 - 1786t + 18,398) \]

Complete the steps below. Show your work.

\[ -0.0005t^2(-3t^3 + 134t^2 - 1786t + 18,398) \]
\[ = (-0.0005t^2)(-3t^3) + (-0.0005t^2)(134t^2) + (-0.0005t^2)(-1786t) + (-0.0005t^2)(18,398) \]
\[ = 0.0015t^5 - 0.067t^4 + 0.893t^3 - 9.199t^2 \]

\[ 0.02t(-3t^3 + 134t^2 - 1786t + 18,398) \]
\[ = (0.02t)(-3t^3) + (0.02t)(134t^2) + (0.02t)(-1786t) + (0.02t)(18,398) \]
\[ = -0.06t^4 + 2.68t^3 - 35.72t^2 + 367.96t \]

\[ 0.23(-3t^3 + 134t^2 - 1786t + 18,398) \]
\[ = (0.23)(-3t^3) + (0.23)(134t^2) + (0.23)(-1786t) + (0.23)(18,398) \]
\[ = -0.69t^3 + 30.82t^2 - 410.78t + 4231.54 \]

Now combine like terms and simplify. Show your work.

\[ (0.0015t^5) + (-0.067t^4 + 0.893t^3 - 0.69t^2) + (-9.199t^2 - 35.72t^2 + 367.96t - 410.78t) + 4231.54 = 0.0015t^5 - 0.127t^4 + 2.883t^3 - 14.099t^2 - 42.82t + 4231.54 \]

What is your function?

\[ y = 0.0015t^5 - 0.127t^4 + 2.883t^3 - 14.099t^2 - 42.82t + 4231.54 \]

7. How many students were in a foreign language class in 2000? Show all your work and use a complete sentence in your answer.

\[ y = 0.0015(20)^5 - 0.127(20)^4 + 2.883(20)^3 - 14.099(20)^2 - 42.82(20) + 4231.54 \]
\[ y = 5279.54 \]

In 2000, approximately 5280 thousand students were in a foreign language class.
Problem 2
Students will evaluate polynomial expressions and will calculate the percent of the total number of students who are taking a Spanish class.

Grouping
Ask for a student volunteer to read Problem 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) through (D) of Problem 2.

Guiding Questions
- What information is given in this problem?
- How does this problem differ from Problem 1? How is this problem similar to Problem 1?
- What are you asked to find in part (A) of Problem 2? How can you find the number of students that were enrolled in a Spanish class in 1990?
- How many years after 1980 was 1990?

Note Students can also complete part (C) by writing and solving a proportion rather than the method suggested in the sample solution.

Call the class back together to have the students discuss and present their work for parts (A) through (D) of Problem 2.

Investigate Problem 2
Grouping
Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Pose the Guiding Questions at the right to verify student understanding. Complete Question 1 together as a whole class.

Problem 2
Spanish, Anyone?
A model for the number of high school students that were in a Spanish class in high school during the years from 1985 to 2000 is \( y = 4t^2 + 14t + 2137 \), where \( t \) represents the number of years since 1980 and \( y \) is the number of high school students in thousands that were in a Spanish class.

A. Find the number of students that were in a Spanish class in 1990. Show your work and use a complete sentence in your answer.
\[
y = 4(10)^2 + 14(10) + 2137 = 2677
\]
In 1990, 2677 thousand high school students were in a Spanish class.

B. Find the total number of students in high school in 1990. You can use the function \( y = -3t^3 + 134t^2 - 1786t + 18,398 \) from Problem 1. Show your work and use a complete sentence in your answer.
\[
y = -3(10)^3 + 134(10)^2 - 1786(10) + 18,398
\]
\[
y = 10,938
\]
In 1990, 10,938 thousand students were in high school.

C. In 1990, what percent of all high school students were in a Spanish class? Show your work and use a complete sentence in your answer.

Sample Answer: 2677 \div 10,938 = 0.24
In 1990, approximately 24 percent of all students were in a Spanish class.

D. Use complete sentences to explain how you found your answer to part (C).

Sample Answer: To find the percent of students that were in a Spanish class, divide the number of students that were in a Spanish class by the total number of students in high school.

Investigate Problem 2
1. Use a complete sentence to explain how you can use the functions given in Problem 2 to write a function that gives the percent of students that were in a Spanish class during the years from 1985 to 2000.

Sample Answer: Divide the function for the number of students that are in a Spanish class by the function for the total number of students in high school.

Guiding Questions
- How did you find the percent of students that were enrolled in Spanish Classes in 1990?
- How would your equation change to allow you to find the percent of students enrolled in Spanish classes for any given year in the domain?
Explore Together

Investigate Problem 2

Students will divide one polynomial by another polynomial.

Take Note

In the quotient $a \div b$, $a$ is the dividend and $b$ is the divisor.

Grouping

Ask for a student volunteer to read Question 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Complete Question 2 together as a whole class working slowly through each step.

Guiding Questions

■ What information is given in this problem?
■ When dividing 3240 by 24, what is the divisor? What is the dividend?
■ To estimate the quotient $3240 \div 24$, we would first think of how many times we could divide 3 by 24 getting 0. Then we would think of how many times we can divide 32 by 24 getting 1. How would you finish dividing 3240 by 24 using long division? Show all of your work.

Take Note

Whenever you divide two polynomials, it is important to write the polynomials in standard form.

Common Student Errors

The process of using long division to divide whole numbers is still confusing to some students. Extending that concept to divide polynomials is significantly more difficult for many students to understand. Try to relate the process to division of whole numbers as best as possible to help students follow the solution in Question 2. When you have finished Question 2, have students take turns explaining each step to verify student understanding.

Investigate Problem 2

2. You can divide two polynomials just like you divide numbers by using long division. For instance, consider the quotient of $10x^3 + 13x^2 + 2x - 4$ and $2x + 1$ shown near the bottom of the page. Begin by finding the quotient of the first terms. In this case, what is $10x^3$ divided by $2x$?

$5x^2$

This is the first term of the quotient. Write this term in the quotient below. Then multiply this expression by the polynomial $2x + 1$. Subtract the result from $10x^3 + 13x^2$ to get $8x^2$ (see Step 1).

Bring down the next term, $2x$, from the dividend. You need to do this because the product of $2x + 1$ and the second term of the quotient will have two terms. Now, we have to divide $8x^2$ by the $2x$ in the divisor. What is the result? This is the second term of the quotient.

$4x$

Write this term in the quotient below. Then multiply this expression by the polynomial $2x + 1$. Subtract the result from $-2x - 4$ (see Step 2 below).

Bring down the last term, $-4$, from the dividend. Finally, we have to divide $-2x$ by $2x$. What is the result? This is the last term of the quotient.

$-1$

Write this term in the quotient below. Then multiply this expression by the polynomial $2x + 1$. Subtract the result from $-2x - 4$ (see Step 3 below).

Because the difference $-3$ is a term whose degree is less than the degree of the divisor ($2x$), we are done. This number $-3$ is the remainder. Write the remainder over the divisor below.

$$\begin{align*}
&\frac{5x^2 + 4x + (-1)}{2x + 1} + \frac{-3}{2x + 1} \\
&\frac{10x^3 + 13x^2 + 2x - 4}{2x + 1} \\
&\frac{-(10x^3 + 5x^2)}{} \quad \text{Step 1} \\
&\frac{8x^2 + 2x}{4x^2 + 4x} \quad \text{Step 2} \\
&\frac{-2x - 4}{-(-2x - 1)} \quad \text{Step 3} \\
&\frac{-3}{-3}
\end{align*}$$
Investigate Problem 2

You can check your answer by multiplying the divisor and quotient and adding the remainder:

<table>
<thead>
<tr>
<th>Divisor</th>
<th>Quotient</th>
<th>Remainder</th>
<th>Dividend</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2x + 1)(5x^2 + 4x - 1))</td>
<td>((-3))</td>
<td></td>
<td>(10x^3 + 13x^2 + 2x - 4)</td>
</tr>
<tr>
<td>(10x^2 + 8x^2 - 2x + 5x + 4x - 1 - 3)</td>
<td>(= 10x^3 + 13x^2 + 2x - 4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10x^2 + 13x^2 + 2x - 4)</td>
<td>(-10x^3 + 13x^2 + 2x - 4)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Find each quotient. Show all your work.

\((x^2 + 6x + 5) - (x + 1) = x + 5\)

\[
\begin{align*}
x + 5 \\
x + 1)x^2 + 6x + 5 \\
-x^2 - x \\
5x + 5 \\
-(5x + 5) \\
0
\end{align*}
\]

\((2x^2 + 5x - 12) - (2x - 3) = x + 4\)

\[
\begin{align*}
x + 4 \\
2x - 3/2x^2 + 5x - 12 \\
-(2x^2 - 3x) \\
8x - 12 \\
-(8x - 12) \\
0
\end{align*}
\]
Investigate Problem 2

Students will continue to divide polynomials using long division.

In Question 4 of Problem 2, students will perform the division for the situation in Problem 2. The students will find an equation to model the percentage of the total number of high school students that are enrolled in a Spanish class during any given year.

Grouping
Students will be working in small groups to complete these questions.

Common Student Errors
Some students incorrectly reverse the divisor and the dividend when using long division to divide whole numbers and will often do the same when dividing polynomials. Be ready to refocus the students and ask them guiding questions to help them recognize their error and divide appropriately.

Arithmetic errors will also be common. As with multiplying polynomials, it will help some students to rewrite the original polynomials in terms of addition rather than subtraction.

Call the class back together to have the students discuss and present their work for Questions 3 and 4.

Notes
Synthetic division for polynomials is not discussed in this lesson. It is too early to present that topic to students who are first learning how to divide polynomials. Synthetic division should be saved until students are very proficient with long division and clearly understand the concepts at work in the process.

Key Formative Assessments
- How can you multiply polynomials?
- How can you use an area model to help multiply polynomials?
- How can you divide polynomials using long division?
- What is a divisor? What is a dividend? What is a remainder?
Wrap Up

Close

- Review all key terms and their definitions. Include the terms *area model, distributive property, divisor, dividend, and remainder*.

- You may also want to review any other vocabulary terms that were discussed during the lesson which may include *percent, percent equation, proportion, decimal form, product, quotient, geometric model, domain, and long division*.

- Remind the students to write the key terms and their definitions in the notes section of their notebooks. You may also want the students to include examples.

- Ask the students to compare and contrast multiplying and dividing polynomials.

- Have the students use the functions from this section to predict the number of students taking a foreign language in high schools this year. Have them do the same for the number of students taking Spanish as their foreign language.

- Have students develop polynomials as a class and list them on the front board. Practice multiplying and dividing some of the pairs of polynomials until it is clear that the students understand the processes.

Follow Up

Assignment

Use the Assignment for Lesson 10.3 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Open-Ended Writing Task

Have the students write a short paragraph explaining how to multiply and how to divide polynomials. Remind them to be precise in their vocabulary. For instance, they should correctly use the words *product, quotient, divisor, dividend, and remainder* in their explanation. They should also include a detailed example of each.
Reflections
Insert your reflections on the lesson as it played out in class today.

What went well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

What did not go as well as you would have liked?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

How would you like to change the lesson in order to improve the things that did not go well and capitalize on the things that did go well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
Lesson Overview

Within the context of this lesson, students will be asked to:

- Multiply binomials.
- Develop patterns and formulas for multiplying binomials.
- Use the FOIL Pattern to multiply binomials efficiently.
- Use formulas to find the products of the most common types of special binomials.

Essential Questions

The following key questions are addressed in this section:

1. What is a binomial?
2. What is the product of two binomials?
3. How can you use the FOIL Pattern to multiply two binomials?
4. What special binomial products are common?
5. What patterns for multiplying binomials exist?
**Warm Up**

Place the following questions or an applicable subset of these questions on the board before students enter class. Students should begin working as soon as they are seated.

Simplify each expression. Where applicable, $x \neq 0$.

1. $3^0 = 1$
2. $(-2)^0 = 1$
3. $-(2)^0 = -1$
4. $-2^0 = -1$
5. $(5x)^2 = 25x^2$
6. $5(x)^2 = 5x^2$
7. $\frac{4x^3}{2x^2} = 2$
8. $-3^3 = -27$
9. $\frac{12x^3}{3x} = 4x^2$
10. $(-2)^5 = -32$
11. $x^5(x^5) = x^7$
12. $(x^5)^5 = x^{10}$
13. $2(x^2)^3 = 2x^6$
14. $-10(2)^3 = -10(8) = -80$
15. $-10(-2)^3 = -10(-8) = 80$
16. $-3(2)^3 = -3(8) = -18$
17. $(4x)^2 = 16x^2$
18. $(-3x^5)^2 = 9x^{10}$

**Motivator**

Begin the lesson with the motivator to get students thinking about the topic of the upcoming problem. This lesson is about an artist making designs in stained glass. The motivating questions are about making stained glass.

Ask the students the following questions to get them interested in the lesson.

- What is stained glass?
- Do you know how stained glass is made?
- Where have you seen stained glass?
- How difficult do you think it is to make stained glass?
Problem 1

Students will represent the area of a rectangular region as the product of two binomials. The students will then multiply the binomials to find the area in a simplified manner.

Grouping

Ask for a student volunteer to read the Scenario and Problem 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Begin by asking the students to complete parts (A) through (C) of Problem 1 individually. Walk around the room to get a sense of how well the students understand the information in the problem. After the students have had a few minutes to think about the situation, call the class back together to discuss and present their work for parts (A) through (C) of Problem 1. Some students may not have been able to complete these correctly on their own.

Guiding Questions

- What information is given in this problem?
- What does “the ratio of the length to the width is 7 to 5” mean?
- If the length of the stained glass is 7 inches, what is the width of the stained glass?
- If the length of the stained glass is 21 inches, what is the width?
- If the length of the stained glass is 21 inches and the width is 15 inches, what is the total length and width of the frame?
- How can you find the area of a rectangle? How can you find the area of the rectangular frame?
- How wide is the wooden frame on each side of the stained glass?

Explore Together

Problem 1

Stained Glass Design

A. Complete the diagram below that models the stained-glass design in its frame.

B. Write an expression for the total length of the stained-glass design with its frame.

\[ 7x + 4 \]

C. Write an expression for the total width of the stained-glass design with its frame.

\[ 5x + 4 \]

D. Write a function for the area of the stained-glass design with the frame.

\[ f(x) = (7x + 4)(5x + 4) \]

E. Use a distributive property to simplify the function in part (D). Show your work.

\[

t(x) = (7x + 4)(5x + 4) \\
= (7x + 4)(5x + 4) \\
= 7x(5x + 4) + 4(5x + 4) \\
= 35x^2 + 20x + 28x + 16 \\
= 35x^2 + 48x + 16 \\

F. What type of function is in part (E)? Use a complete sentence in your answer.

The function is a quadratic function.

Guiding

Have students work together in small groups to complete parts (D) through (F). Then call the class back together to have the students discuss and present their work for parts (D) through (F) of Problem 1.
Explore Together

Investigate Problem 1

Students will investigate multiplication of two binomials using the FOIL pattern.

Grouping

Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Complete Question 1 together as a whole class.

Guiding Questions

- What is standard form?
- What kind of function has a degree of 1?
- If a linear function has a positive coefficient, is it an increasing or decreasing function? What will the graph look like?
- If a linear function has a negative coefficient, is it an increasing or decreasing function? What will the graph look like?
- What kind of function has a degree of 2?
- If a quadratic function has a positive coefficient, what will the graph of the function look like?
- If a quadratic function has a negative coefficient, what will the graph of the function look like?
- What kind of function has a degree of 3?

Grouping

Ask for a student volunteer to read Question 2 aloud. Have a student restate the problem. Pose the Guiding Questions at the right to verify student understanding, then solve Question 2 together as a whole class. Have students work together in small groups to complete Question 3, then call the class back together to have the students discuss and present their work for Question 3.

Investigate Problem 1

1. What is the value of writing the function in standard form in part (E)? Show your work and use a complete sentence in your answer.

Sample Answer: You can tell whether the graph, a parabola, opens upward or downward. You can also easily determine the y-intercept and the vertex.

2. In part (E), you found the product of two binomials by using the distributive property. Consider the product after two uses of the distributive property:

\[(2x + 3)(x + 8) = (2x + 3)(x + 3)(8)\]

First use of distributive property

\[(2x + 3)(x + 8) = 2x(x) + 2x(8) + 3(x) + 3(8)\]

Second use of distributive property

You can quickly find the product of two binomials by remembering the word FOIL. FOIL stands for multiplying the first terms, the outer terms, the inner terms, and the last terms. Then the products are added together as shown above. Finish finding the product of \(2x + 3\) and \(x + 8\).

\[2x(x) + 2x(8) + 3(x) + 3(8) = 2x^2 + 16x + 3x + 24\]

\[= 2x^2 + 19x + 24\]

3. Use the FOIL pattern to find each product. Show your work.

\[(x + 2)(x + 9)\]

\[(x + 2)(x + 9) = x^2 + 9x + 2x + 18\]

\[= x^2 + 11x + 18\]

\[(x - 3)(x - 6)\]

\[(x - 3)(x - 6) = x^2 - 6x - 3x + 18\]

\[= x^2 - 9x + 18\]

\[(3x - 4)(x + 2)\]

\[(3x - 4)(x + 2) = 3x^2 + 6x - 4x - 8\]

\[= 3x^2 + 2x - 8\]

Guiding Questions

- What is the distributive property?
- What is distributed in the first step of the example in Question 2? What is distributed in the second step?
Problem 2

Students will work to develop a pattern for squaring a binomial.

Grouping

Ask for a student volunteer to read Problem 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) through (D) of Problem 2.

Guiding Questions

■ What information is given in this problem?
■ How is this problem different from Problem 1? How are the problems the same?
■ How can you calculate the area of a rectangle? How can you calculate the area of a square?
■ What is the length of a side of the stained glass in Problem 2? What is the width of the wooden frame?
■ What is the length of each side of the stained glass including the frame?

Investigate Problem 2

Just the Math

Students will identify the pattern for squaring a binomial. This pattern is a time saving tool that will reduce tedium and frustration for students. Squaring values is an important skill used in many applications for problem solving.

Grouping

Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Complete Question 1 together as a whole class. Ask for a
Investigate Problem 2

Students will continue to investigate the pattern for squaring a binomial.

Grouping
Students will be working together in small groups to complete Question 2.

Common Student Errors
Students may have difficulty recognizing the patterns in squaring a binomial. If students get frustrated with this you may want to call the class back together to pose the Guiding Questions below to help the students better understand the question. You may also want to work together as a whole class to complete Question 2.

Guiding Questions
■ What would correctly complete $(x + 5)^2 = (\underline{\quad})^2 + (\underline{\quad})^2$?
■ How is $(x + 5)(x + 5)$ special?
■ What are the outside terms? What is their product?
■ What are the inside terms? What is their product?
■ What do you notice about the inside and outside terms and their products?
■ What are the first terms? What are the last terms?
■ How are the first and the last terms special? How is each of their products special?

Call the class back together to have the students discuss and explain their work for Question 2.

Grouping
Ask for a student volunteer to read Question 3 aloud. Have a student restate the problem. Have students work together in small groups to complete Questions 3 and 4.

Investigate Problem 2

Does this relationship hold true for the square of the binomials in parts (C) and (D)? Use complete sentences to explain.

Yes, this is true for $(x + 6)(x + 6)$, because the middle term $12x$ is two times the product of the linear term $x$ and the constant term $6$. This is also true for $(x + 10)(x + 10)$, because the middle term $20x$ is two times the product of the linear term $x$ and the constant term $10$.

In $(x + 3)(x + 3)$, which is $x^2 + 6x + 9$, how is the last term 9 related to the constant term 3? Use a complete sentence to explain.

The last term 9 is the square of the constant term 3.

Does this relationship hold true for the square of the binomials in parts (C) and (D)? Use complete sentences to explain.

Yes, this is true for $(x + 6)(x + 6)$, because the last term 36 is the square of the constant term 6. This is also true for $(x + 10)(x + 10)$, because the last term 100 is the square of the constant term 10.

Complete the formula below for simplifying the square of a binomial sum of the form $(a + b)^2$. Then use the FOIL pattern to check your answer.

$(a + b)^2 = (\underline{\quad})^2 + 2(\underline{\quad})(\underline{\quad}) + (\underline{\quad})^2$

FOIL:

$$(a + b)^2 = (a + b)(a + b)$$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

3. Use the FOIL pattern to find the products $(x - 4)^2$ and $(4x - 3)^2$. Show your work.

$$(x - 4)^2 = (x - 4)(x - 4)$$

$= x^2 - 4x - 4x + 16$

$= x^2 - 8x + 16$

$$(4x - 3)^2 = (4x - 3)(4x - 3)$$

$= 16x^2 - 12x - 12x + 9$

$= 16x^2 - 24x + 9$

Guiding Questions
■ What is the FOIL pattern and how is it used?
■ Complete the statement: $(x - 4) = (x + \underline{\quad})$.  

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Explore Together

Investigate Problem 2

Students will investigate various applications of the FOIL pattern.

Grouping

Students will be working together in small groups to complete Questions 3 and 4.

Common Student Errors

Students will often confuse $(5x)^2$ with the expression $5x^2$. Remind students of the difference between these expressions. If necessary, refer back to the Warm Up for this lesson to refresh the students’ understanding of this relationship.

Call the class back together to have the students discuss and present their work for Questions 3 and 4.

Grouping

Ask for a student volunteer to read Question 5 aloud. Have a student restate the problem. Then have the students complete Questions 5 and 6 individually.

Call the class back together to have the students discuss and present their work for Questions 5 and 6.

Have students work together in small groups to complete Question 7, then call the class back together to have the students discuss and present their work for Question 7.

Key Formative Assessments

- In your own words, describe the term FOIL pattern.
- How can you use the FOIL pattern?
- How can you find the square of a binomial sum?
- How can you find the square of a binomial difference?
- What other FOIL pattern(s) did you discover?
- How is the FOIL pattern related to the distributive property?

---

Investigate Problem 2

4. Complete the formula for simplifying the square of a binomial difference of the form $(a - b)^2$. Then use the FOIL pattern to show that your formula works.

$$(a - b)^2 = a^2 - 2ab + b^2$$

FOIL:

$$\begin{align*}
(a - b)^2 &= (a - b)(a - b) \\
&= a^2 - ab - ab + b^2 \\
&= a^2 - 2ab + b^2
\end{align*}$$

5. Use the FOIL pattern to find the products $(x - 3)(x + 3)$ and $(2x - 5)(2x + 5)$. Show your work.

$$(x - 3)(x + 3) = x^2 + 3x - 3x - 9 = x^2 - 9$$

$$(2x - 5)(2x + 5) = 4x^2 + 10x - 10x - 25 = 4x^2 - 25$$

6. Write a formula for simplifying the product of a sum and a difference of the form $(a - b)(a + b)$. Then use the FOIL pattern to show that your formula works.

$$\begin{align*}
(a - b)(a + b) &= a^2 - b^2 \\
\text{FOIL:} \\
(a - b)(a + b) &= a^2 + ab - ab - b^2 \\
&= a^2 - b^2
\end{align*}$$

7. Find each product. Show your work.

$$\begin{align*}
(x + 5)(x + 3) &= x^2 + 3x + 5x + 15 \\
&= x^2 + 8x + 15 \\
(3x + 4)(x - 2) &= 3x^2 - 6x + 4x - 8 \\
&= 3x^2 - 2x - 8 \\
(2x + 1)^2 &= (2x)^2 + 2(2x)(1) + 1 \\
&= 4x^2 + 4x + 1 \\
(x + 9)(x - 9) &= x^2 - 81 \\
(3x + 2)(3x - 2) &= 9x^2 - 4
\end{align*}$$
Close

- Review all key terms and their definitions. Include the terms FOIL pattern, square of a binomial sum, and square of a binomial difference.
- You may also want to review any other vocabulary terms that were discussed during the lesson which may include sum, difference, polynomial, monomial, binomial, trinomial, distributive property, standard form, coefficient, square, and cube.
- Remind the students to write the key terms and their definitions in the notes section of their notebooks. You may also want the students to include examples.
- Pose the following questions for the students.
  - Summarize the FOIL pattern.
  - What do the letters in FOIL represent?
  - How can you find the square of a binomial sum?
  - How can you find the square of a binomial difference?
  - What binomials give a product that is the difference of two squares, such as $x^2 - 4$?
  - Why is it easier to use the FOIL pattern than to use the distributive property to multiply two binomials?
  - Can you use the FOIL pattern to multiply two trinomials? Why or why not?

Follow Up

Assignment

Use the Assignment for Lesson 10.4 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Open-Ended Writing Task

Have the students explain in a detailed, but short paragraph, the steps to use to multiply by using FOIL.
Reflections
Insert your reflections on the lesson as it played out in class today.

What went well?

_______________________________________________________________________________________________
_______________________________________________________________________________________________

What did not go as well as you would have liked?

_______________________________________________________________________________________________
_______________________________________________________________________________________________

How would you like to change the lesson in order to improve the things that did not go well and capitalize on the things that did go well?

_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________

Notes
Lesson 10.5
Suspension Bridges
Factoring Polynomials

Learning By Doing Lesson Map

Objectives
In this lesson, you will:
■ Factor a polynomial by factoring out a common factor.
■ Factor a polynomial of the form \( x^2 + bx + c \).
■ Factor a polynomial of the form \( ax^2 + bx + c \).

Key Terms
■ factor
■ linear factor
■ trinomial
■ FOIL pattern

Sunshine State Standards
MA.912.A.4.3
Factor polynomial expressions.

MA.912.A.10.1
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

MA.912.A.10.2
Decide whether a solution is reasonable in the context of the original situation.

Lesson Overview
Within the context of this lesson, students will be asked to:
■ Solve a quadratic equation using the quadratic formula.
■ Factor out a common monomial factor to solve a quadratic function.
■ Factor trinomials of the form \( x^2 + bx + c \).
■ Factor trinomials of the form \( ax^2 + bx + c \).

Essential Questions
The following key questions are addressed in this section:
1. What is a suspension bridge?
2. What is a factor?
3. What is a linear factor?
4. How can you factor a trinomial of the form \( x^2 + bx + c \)?
5. How can you factor a trinomial of the form \( ax^2 + bx + c \)?
Warm Up
Place the following questions or an applicable subset of these questions on the board before students enter class. Students should begin working as soon as they are seated.

Find the greatest common factor of each set of numbers.

1. 63 and 72  9
2. 18 and 24  6
3. 75 and 250  25
4. 15, 45, and 80  5

Classify each polynomial as a monomial, a binomial, or a trinomial.

5. 2x + 3  binomial
6. 8  monomial
7. x² − 2x − 4  trinomial
8. 5x  monomial
9. x − 9  binomial

Multiply each binomial by using the FOIL pattern.

10. (x + 2)(x + 5)  x² + 7x + 10
11. (x − 3)(x + 1)  x² − 2x − 3
12. (x + 4)(x − 4)  x² − 16
13. (x − 8)(x − 8)  x² − 16x + 64

Motivator
Begin the lesson with the motivator to get students thinking about the topic of the upcoming problem. This lesson is about preparing a presentation about suspension bridges. The motivating questions are about bridges.

Ask the students the following questions to get them interested in the lesson.

■ What is a suspension bridge?
■ What other kinds of bridges have been constructed?
■ Where are the nearest bridges to our school?
■ What materials do you think are used to create most bridges?
Lesson 10.5  ■  Factoring Polynomials

**Problem 1**

Students will use the quadratic formula to solve a quadratic equation.

**Guiding Questions**

- What information is given in this problem?
- What is a suspension bridge?
- What does the phrase “high water level” mean?
- What does \( y \) represent in this situation? What is the unit for \( y \)?
- What does \( x \) represent in this situation? What is the unit for \( x \)?
- How far above the high water level is the suspension cable attached to the left tower on the bridge?
- How can you find the height above the high water level at a point 10 meters to the right of the left tower?
- How many times between the left and right tower do you expect the cable to be 90 meters above the high water level? Why?
- What is the highest point that the cable will be above the high water level?

**Common Student Errors**

Students may have difficulty recognizing the need to use the quadratic formula to complete part (C). You may also have to remind students that the quadratic formula is

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

and that the formula is used when the quadratic equation is written in the form \( ax^2 + bx + c = 0 \).

**SCENARIO**

In your science class, you have to make a presentation on a “feat of engineering.” You have decided to make your presentation on suspension bridges, such as the one shown below. During the research for the presentation, you have discovered that the cables that are used to carry the weight on a bridge form the shape of a parabola. Because you have recently studied parabolas in your math class, you decide to use what you know about parabolas in your presentation.

**Problem 1  Modeling the Cable Shape**

You have modeled the main cable’s shape of a particular suspension bridge by using the function \( y = 0.002x^2 - 0.4x + 96 \), where \( x \) is the distance in meters from the left tower and \( y \) is the height in meters of the cable above the high water level.

**A.** Write and simplify an equation that you can use to find the cable’s horizontal distance from the left tower at 96 meters above the high water level. Show your work.

\[
96 = 0.002x^2 - 0.4x + 96
\]

\[
0 = 0.002x^2 - 0.4x
\]

**B.** How can you solve the equation in part (A)? Use a complete sentence in your answer.

Sample Answer: You can solve the equation by using the Quadratic Formula.

**C.** Solve the equation in part (A). Show all your work.

\[
x = \frac{-(-0.4) \pm \sqrt{(-0.4)^2 - 4(0.002)(0)}}{2(0.002)}
\]

\[
x = \frac{0.4 \pm 0.4}{0.004}
\]

\[
x = \frac{0.4 + 0.4}{0.004} = \frac{0.8}{0.004} = 200 \text{ or } x = \frac{0.4 - 0.4}{0.004} = 0
\]
Problem 1

Students will investigate factors of a polynomial.

Grouping

Students will be working together in small groups to complete parts (A) through (D) of Problem 1.

Call the class back together to have the students discuss and present their work for parts (A) through (D) of Problem 1.

Key Formative Assessments

- What does an x-value of zero represent in this situation?
- What does an x-value of 200 represent in this situation?
- How far apart are the left and right towers?

Investigate Problem 1

Grouping

Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Question 1.

Guiding Questions

- What is a quadratic function?
- What other types of functions have you learned?
- What is the degree of a quadratic function? What is the degree of a linear function? What is the degree of a cubic function?
- How can you solve the equation $3x = 12$ for $x$?
- Divide each side of the equation $3x - 12 = 0$ by 3. What is the result? If $3x - 12 = 0$, is it also true that $x - 4 = 0$?

Problem 1: Modeling the Cable Shape

D. Interpret your solutions from part (C) in the problem situation. Use a complete sentence in your answer. When the cable is at a height of 96 feet, the cable is 0 feet and then 200 feet from the left tower of the bridge.

Investigate Problem 1

1. In the last lesson, you multiplied polynomials. The product of what kinds of polynomials gives you a quadratic (second-degree) polynomial? Use a complete sentence in your answer.

Sample Answer: The product of two linear polynomials is a quadratic polynomial.

Consider your equation again from part (A):

$$0.002x^2 - 0.4x = 0$$

Rewrite your equation so that each term is written as a product with this common expression as a factor.

$$x(x - 200) = 0$$

The product of what two expressions gives you $x^2 - 200x$?

$x$ and $x = 200$

So now you have the following.

$$x^2 - 200x = 0$$
$$x(x - 200) = 0$$
$$x(200 - x) = 0$$

What are the solutions of the equation?

$x = 0$ and $x = 200$

- Why might we prefer to work with the equation $x - 4 = 0$ than with the equation $3x - 12 = 0$?

Common Student Errors

Question 1 may be confusing for some students. If many students struggle, you may want to complete Question 1 together as a whole class.
Investigate Problem 1

Students will be introduced formally to factoring polynomials.

Call the class back together to have the students discuss and present their work for Question 1.

Just the Math

Students will factor out common monomial factors from polynomials.

Take Note

Recall the Symmetric Property from Lesson 4.4: For any real numbers \(a\) and \(b\), if \(a = b\), then \(b = a\).

Grouping

Ask for a student volunteer to read Question 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Questions 2 and 3.

Take Note

An expression is factored completely when none of the factors of the expression can be factored further.

Guiding Questions

- What is a common factor?
- What are the common factors of 15 and 45? What is the greatest common factor of 15 and 45?
- What is the common factor in the function \(x^2 - 200x\)?
- Complete each of the following multiplicative distributive properties.

\[
\begin{align*}
ab + ac &= a(b + c) \\
ab - ac &= a(b - c) \\
(b + c)a &= ba + ca \\
(b - c)a &= ba - ca
\end{align*}
\]

Investigate Problem 1

You have just solved the equation by factoring the polynomial. How do these solutions compare to the solutions from part (C)? Use a complete sentence in your answer.

They are the same.

Which method of finding the solutions was easier? Why? Use a complete sentence in your answer.

Sample Answer: The factoring method was easier because it took fewer steps to find the answer.

2. Just the Math: Factoring Out a Common Factor

In Question 1, you solved the equation by factoring out a common factor of \(x\). Which mathematical rule was used to factor the polynomial? Use a complete sentence to explain your reasoning.

Sample Answer: The distributive property \(a(b - c) = ab - ac\) was used, just in the reverse direction.

So, because the equality is symmetric, you can write your multiplicative distributive properties as follows:

\[
\begin{align*}
ab + ac &= a(b + c) \\
ab - ac &= a(b - c) \\
ba + ca &= (b + c)a \\
ba - ca &= (b - c)a
\end{align*}
\]

When the distributive properties are used in this manner, you are factoring expressions.

Factor each expression completely. Show your work.

\[
\begin{align*}
x^2 + 12x &= x(x) + x(12) \\
x^2 - 5x &= x(x^2) - x(5)
\end{align*}
\]

\[
\begin{align*}
3x^2 - 9x &= 3x(x) - 3x(3) \\
10x^2 - 6 &= 2(5x^2) - 2(3)
\end{align*}
\]

3. What is the domain of the function in Problem 1?

Use complete sentences to explain your reasoning.

Sample Answer: The domain is all real numbers from 0 to 200. Because the cable's shape is a parabola and the towers are the same height, the cable height should be the same at both towers. The cable starts 0 meters from the left tower and is at a height of 96 meters and the cable again reaches a height of 96 meters at 200 meters from the left tower.

Notes

You may want to ask classes of advanced students to calculate the vertex for the bridge situation. When they find the \(x\)-value of 100 for the vertex, they can substitute into the function to get the range for \(y\)-values to be between 76 and 96 meters inclusive.
Problem 2
Students will continue to investigate factoring polynomials.

Grouping
Ask for a student volunteer to read Problem 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete parts (A) through (C) of Problem 2.

Guiding Questions
What is the original function relating the horizontal distance from the left tower and the height of the cable above the high water level?
What variable is equal to 81 meters in Problem 2?

Call the class back together to have the students discuss and present their work for parts (A) through (C) of Problem 2.

Investigate Problem 2
Ask for a student volunteer to read Question 1 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Question 1.

Guiding Questions
What is the product of \((x + a)(x + b)\) using the FOIL pattern?
What is the polynomial that you wrote in part (A) of Problem 2?
What is a product? What is a sum?
Is the product of 2 positive numbers positive or negative? Is the product of two negative numbers positive or negative?

Problem 2 Using the Cable Shape Model
A. Write and simplify an equation that you can use to find the cable’s horizontal distance from the left tower at 81 meters above the high water level. Show your work.
\[
81 = 0.002x^2 - 0.4x + 96
0 = 0.002x^2 - 0.4x + 15
\]
Now divide each side by 0.002.
\[
0 = x^2 - 200x + 7500
\]
B. How is this equation the same as or different from the equation in Problem 1? Use a complete sentence in your answer.
Sample Answer: The polynomial in this equation has a constant term. Both polynomials have the same \(x^2\)-term and \(x\)-term.
C. Can you factor out a common factor?
No.

Investigate Problem 2
1. If we want to factor the polynomial from part (A) as the product of two linear expressions \((x + a)(x + b)\), what must be true about the product \(ab\)? Explain your reasoning. Use a complete sentence in your answer.
Sample Answer: The product \(ab\) must be 7500 because \((x + a)(x + b) = x^2 + bx + ax + ab\) and we want \(x^2 + bx + ax + ab = x^2 - 200x + 7500\).
What must be true about the sum \(a + b\)? Explain your reasoning. Use a complete sentence in your answer.
Sample Answer: The sum \(a + b\) must be \(-200\) because \((x + a)(x + b) = x^2 + bx + ax + ab\) and we want \(x^2 + bx + ax + ab = x^2 - 200x + 7500\).

Make a list of pairs of negative integers whose product is 7500. Then find the sum of each pair of numbers. Stop when you find the pair of numbers that has a sum of \(-200\).

\[
\begin{align*}
-1, -7500; & \quad \text{Sum: } -7501 \\
-2, -3750; & \quad \text{Sum: } -3752 \\
-3, -2500; & \quad \text{Sum: } -2503 \\
-4, -1875; & \quad \text{Sum: } -1879 \\
-5, -1500; & \quad \text{Sum: } -1505 \\
-6, -1250; & \quad \text{Sum: } -1256 \\
-10, -750; & \quad \text{Sum: } -760 \\
-12, -625; & \quad \text{Sum: } -637 \\
-15, -500; & \quad \text{Sum: } -515 \\
-20, -375; & \quad \text{Sum: } -395 \\
-25, -300; & \quad \text{Sum: } -325 \\
-30, -250; & \quad \text{Sum: } -280 \\
-50, -150; & \quad \text{Sum: } -200
\end{align*}
\]
Explore Together

Investigate Problem 2

Students will investigate factoring polynomials of the form \(x^2 + bx + c\).

Grouping

Students will be working together in small groups to complete Question 1.

Call the class back together to have the students discuss and present their work for Question 1.

Key Formative Assessments

- What is the product of any number and zero?
- If the product of two numbers is zero, what do we know about at least one of the factors?
- Why did we set the factors of \((x + (-50))\) and \((x + (-150))\) equal to zero in Question 1?

Take Note

Your special product rules from Lesson 10.4 can help you factor special products:

- \(a^2 + 2ab + b^2 = (a + b)^2\)
- \(a^2 - 2ab + b^2 = (a - b)^2\)
- \(a^2 - b^2 = (a + b)(a - b)\)

Grouping

Ask for a student volunteer to read Question 2 aloud. Have a student restate the problem. Pose the Guiding Questions below to verify student understanding. Have students work together in small groups to complete Questions 2 through 4.

Guiding Questions

- How can you factor a trinomial of the form \(ax^2 + bx + c\)?
- How can you factor a trinomial of the form \(ax^2 - bx + c\)?
- How can you factor a trinomial of the form \(ax^2 - bx - c\)?
- How can you verify your factorization using the FOIL pattern?

Investigate Problem 2

Which pair of numbers has a product of 7500 and a sum of −200?

-50 and −150

Complete the factorization of \(x^2 - 200x + 7500\).

\[
x^2 - 200x + 7500 = (x + (-50))(x + (-150))
\]

What are the solutions of the equation \(x^2 - 200x + 7500 = 0\)? Show your work.

\[
x - 50 = 0 \text{ or } x - 150 = 0
\]

\[
x = 50 \quad x = 150
\]

What is the cable’s horizontal distance from the left tower at 81 meters above high water level? Use a complete sentence in your answer.

The cable’s horizontal distances from the left tower at 81 meters above high water level are 50 meters and 150 meters.

2. Factor each trinomial as a product of linear factors. Then use the FOIL pattern to verify your answer. Show your work.

\[
\begin{align*}
x^2 + 4x + 3 &= (x + 3)(x + 1) \\
x^2 - 4x - 5 &= (x - 5)(x + 1) \\
x^2 - x - 6 &= (x - 3)(x + 2) \\
x^2 - 9x + 20 &= (x - 4)(x - 5) \\
x^2 + 18x + 81 &= (x + 9)(x + 9) \\
x^2 - 36 &= (x + 6)(x - 6)
\end{align*}
\]

3. How does the sign of the constant in the trinomial determine the signs of the constants in the linear factors? Use complete sentences in your answer.

Sample Answer: If the trinomial constant is positive, then the constants in the factors are either both positive or both negative. If the trinomial constant is negative, then in the factors, one constant is positive and one is negative.
Explore Together

Investigate Problem 2

Students will investigate factoring of polynomials of the form $ax^2 + bx + c$.

Grouping

Students will be working together in small groups to complete Questions 2 through 4.

Call the class back together to have the students discuss and present their work for Questions 2 through 4.

Grouping

Ask for a student volunteer to read Question 5 aloud. Have a student restate the problem. Complete Question 5 together as a whole class. Ask for a student volunteer to read Question 6 aloud. Have a student restate the problem. Have students work together in small groups to complete Question 6.

Call the class back together to have the students discuss and present their work for Question 6.

Key Formative Assessments

- What is a common factor?
- Can a common factor be a constant?
- Can a common factor be a monomial expression?
- How can you factor out a common monomial factor from a polynomial?
- How can you factor a trinomial of the form $x^2 + bx + c$?
- How can you factor a trinomial of the form $ax^2 + bx + c$?
- How can you factor a trinomial of the form $ax^2 - bx + c$?
- Are parentheses necessary for writing a factorization?

Investigate Problem 2

4. In Questions 1 and 2, what do all the trinomials have in common? How are they different? Use complete sentences in your answer.

Sample Answer: All the coefficients of the $x^2$-terms are 1 and the coefficients of the other terms are different.

5. Consider the trinomial $2x^2 + 11x + 9$. If we want to factor this expression as a product of linear factors, what must be true about the coefficients of the $x$-terms of the linear factors? Use complete sentences to explain your reasoning.

Sample Answer: One of the $x$-terms must have a coefficient of one and the other must have a coefficient of two because the coefficient of the product of these $x$-terms must be 2.

What must be true about the constants of the linear factors? Use complete sentences to explain your reasoning.

Sample Answer: Because the product of the constants must be nine, the constants could be 1 and 9 or 3 and 3.

Now that you have identified all the possible coefficients of the $x$-terms and the constants for the linear factors, list the possible pairs of linear factors below that could result from the different positions of the numbers in the factors. The first one is listed.

$$(2x + 3)(1x + 3) = 2x^2 + 9x + 9$$

$$(2x + 1)(1x + 9) = 2x^2 + 19x + 9$$

$$(2x + 9)(1x + 1) = 2x^2 + 11x + 9$$

Now find the product of each pair of linear factors and write your answer above. Which pair of linear factors gives the factorization of $2x^2 + 11x + 9$?

$$2x^2 + 11x + 9 = (2x + 9)(1x + 1)$$

6. Factor each trinomial as a product of linear factors. Then use the FOIL pattern to verify your answer. Show your work.

$$3x^2 + 7x + 2 = [3x + 1][x + 2]$$

$$(3x + 1)(x + 2) = 3x^2 + 6x + x + 2$$

$$= 3x^2 + 7x + 2$$

$$5x^2 + 17x + 6 = [5x + 2][x + 3]$$

$$(5x + 2)(x + 3) = 5x^2 + 15x + 2x + 6$$

$$= 5x^2 + 17x + 6$$

$$6x^2 + 19x + 10 = [3x + 2][2x + 5]$$

$$(3x + 2)(2x + 5) = 6x^2 + 15x + 4x + 10$$

$$= 6x^2 + 19x + 10$$

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Close

- Review all key terms and their definitions. Include the terms factor, linear factor, trinomial, and FOIL pattern.

- You may also want to review any other vocabulary terms that were discussed during the lesson which may include feat of engineering, quadratic formula, standard form, degree of a polynomial, linear function, quadratic function, distributive property, and symmetric property.

- Remind the students to write the key terms and their definitions in the notes section of their notebooks. You may also want the students to include examples.

- Pose the following questions for the students.
  1. How can you factor out a common monomial factor from a polynomial?
  2. How can you factor a polynomial of the form $x^2 + bx + c$? How can you factor a polynomial of the form $ax^2 + bx + c$?
  3. How are factoring a polynomial of the form $x^2 + bx + c$ and factoring a polynomial of the form $ax^2 + bx + c$ different? How are they the same?
  4. How can you use the factored form of a trinomial to solve an equation?
  5. It is easier to solve a quadratic equation using the quadratic formula or the factored form of the equation? Explain your reasoning.
  6. When would you use the quadratic formula to solve a quadratic equation rather than factoring?

Follow Up

Assignment

Use the Assignment for Lesson 10.5 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Open-Ended Writing Task

Ask the students to respond to the following writing prompt. In Question 2 of Investigate Problem 1, you factored out common monomial factors like the $3x$ from each term of the function $(3x^2 - 9x)$. In Question 2 of Investigate Problem 2, you factored out binomial factors like $(x + 3)$ from the trinomial $(x^2 + 4x + 3)$. How can you combine these skills to factor the polynomial $(2x^3 + 16x^2 + 30x)$?
Reflections
Insert your reflections on the lesson as it played out in class today.

What went well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

What did not go as well as you would have liked?
_______________________________________________________________________________________________
_______________________________________________________________________________________________

How would you like to change the lesson in order to improve the things that did not go well and capitalize on the things that did go well?
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
_______________________________________________________________________________________________
More Factoring
Factoring Polynomial Expressions

Objectives
In this lesson, you will:
- Factor perfect square trinomials.
- Factor the difference of two squares.
- Factor polynomial expressions using greatest common factor.
- Factor polynomial expressions by grouping.

Key Terms
- difference of two squares
- perfect square trinomial
- polynomial expression
- greatest common factor
- factoring by grouping

Sunshine State Standards
MA.912.A.10.1
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

MA.912.A.10.2
Decide whether a solution is reasonable in the context of the original situation.

Essential Ideas
- Differences of two squares are special polynomials written in the form \(a^2 - b^2 = (a + b)(a - b)\), where \(a\) and \(b\) are any real numbers.
- Perfect square trinomials are special polynomials written in the form \(a^2 + 2ab + b^2 = (a + b)^2\) or \(a^2 - 2ab + b^2 = (a - b)^2\), where \(a\) and \(b\) are any real numbers.
- A polynomial expression is an expression written in the form \(a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0\), where \(a_0, a_1, \ldots, a_n\) are any real numbers and \(n\) is a positive integer.
- In a polynomial expression, the (non-zero) term having the highest power determines the degree of the polynomial expression.
- The greatest common factor of a polynomial is the largest factor that is common to all terms of the polynomial.
- To factor by grouping, create two groups of terms and factor the greatest common factor of each group. Then factor the greatest common factor of the groups.

Essential Questions
1. What are differences of two squares?
2. What are perfect square trinomials?
3. What is a polynomial expression?
4. What determines the degree of a polynomial expression?
5. What is the greatest common factor of a polynomial?
6. What does “factor by grouping” mean?
Warm Up

Use a graphing calculator to graph each of the following quadratic functions.

1. \( f(x) = x^2 - 1 \)
   How many solutions or zeros? What are they?
   **Two solutions:** 1, -1
   Factor the quadratic equation.
   \( f(x) = (x + 1)(x - 1) \)

2. \( f(x) = x^2 - 4 \)
   How many solutions or zeros? What are they?
   **Two solutions:** 2, -2
   Factor the quadratic equation.
   \( f(x) = (x + 2)(x - 2) \)

3. \( f(x) = x^2 - 9 \)
   How many solutions or zeros? What are they?
   **Two solutions:** 3, -3
   Factor the quadratic equation.
   \( f(x) = (x + 3)(x - 3) \)

4. \( f(x) = x^2 - 16 \)
   How many solutions or zeros? What are they?
   **Two solutions:** 4, -4
   Factor the quadratic equation.
   \( f(x) = (x + 4)(x - 4) \)

5. \( f(x) = x^2 - 25 \)
   How many solutions or zeros? What are they?
   **Two solutions:** 5, -5
   Factor the quadratic equation.
   \( f(x) = (x + 5)(x - 5) \)

6. Without graphing \( f(x) = x^2 - 36 \), how many solutions or zeros and what are they?
   **Two solutions:** 6, -6
   Factor the quadratic equation.
   \( f(x) = (x + 6)(x - 6) \)
Motivator

Use a graphing calculator to graph each of the following quadratic functions.

- \( f(x) = x^2 + 2x + 1 \)
  
  How many solutions or zeros? What are they?
  
  One solution: \(-1\)

- \( f(x) = (x + 1)(x + 1) \)
  
  Factor the quadratic equation.

- \( f(x) = x^2 + 4x + 4 \)
  
  How many solutions or zeros? What are they?
  
  One solution: \(-2\)

- \( f(x) = (x + 2)(x + 2) \)

- \( f(x) = x^2 + 6x + 9 \)
  
  How many solutions or zeros? What are they?
  
  One solution: \(-3\)

- \( f(x) = (x + 3)(x + 3) \)

- \( f(x) = x^2 + 8x + 16 \)
  
  How many solutions or zeros? What are they?
  
  One solution: \(-4\)

- \( f(x) = (x + 4)(x + 4) \)

- \( f(x) = x^2 + 10x + 25 \)
  
  How many solutions or zeros? What are they?
  
  One solution: \(-5\)

- \( f(x) = (x + 5)(x + 5) \)

- Without graphing \( f(x) = x^2 + 12x + 36 \), how many solutions or zeros and what are they?
  
  One solution: \(-6\)

- \( f(x) = (x + 6)(x + 6) \)
### Problem 1

**Differences of two squares** are special polynomials written in the form $a^2 - b^2 = (a + b)(a - b)$, where $a$ and $b$ are any real numbers. **Perfect square trinomials** are special polynomials written in the form $a^2 + 2ab + b^2 = (a + b)^2$ or $a^2 - 2ab + b^2 = (a - b)^2$, where $a$ and $b$ are any real numbers. Students are given several problems of each type and asked to distinguish between and write them in their factored form.

### Grouping

Have students work in groups to complete Problem 1.

#### Explore Together

**Problem 1**

1. **Multiply each pair of binomials.**
   - a. $(x - 4)(x + 4) = x^2 + 4x - 4x - 16 = x^2 - 16$
   - b. $(x + 3)(x + 5) = x^2 + 3x + 5x + 15 = x^2 + 8x + 15$
   - c. $(x - 3)(x + 3) = x^2 - 3x + 3x - 9 = x^2 - 9$
   - d. $(x - 6)(x - 6) = x^2 - 6x - 6x + 36 = x^2 - 12x + 36$
   - e. $(x - 7)^2 = x^2 - 7x - 7x + 49 = x^2 - 14x + 49$

2. **Factor each polynomial.**
   - a. $x^2 - 4x + 4 = (x - 2)(x - 2) = (x - 2)^2$
   - b. $x^2 - 25 = (x - 5)(x + 5)$
   - c. $x^2 + 10x + 25 = (x + 5)(x + 5) = (x + 5)^2$
   - d. $x^2 + 4x + 4 = (x + 2)(x + 2) = (x + 2)^2$
   - e. $4x^2 - 49 = (2x - 7)(2x + 7)$

The expressions in Problem 1 are examples of two special polynomials. One type is the **difference of two squares**. The other type is a **perfect square trinomial**.

3. **Which expressions in Questions 1 and 2 are examples of the difference of two squares?** Write each polynomial and its factored form.
   - $x^2 - 4x + 4 = x^2 - 16$
   - $(x - 3)(x + 3) = x^2 - 9$
   - $x^2 - 25 = (x - 5)(x + 5)$
   - $4x^2 - 49 = (2x - 7)(2x + 7)$

4. **What do you notice about the factors of a polynomial that is the difference of two squares?**
   - The factors of a polynomial that is the difference of two squares involve the same terms but the opposite sign.

5. **Which expressions in Questions 1 and 2 are examples of perfect square trinomials?** Write each polynomial and its factored form.
   - $(x + 4)(x + 4) = x^2 + 8x + 16$
   - $(x - 6)(x - 6) = x^2 - 12x + 36$
   - $(x - 7)^2 = x^2 - 14x + 49$
   - $x^2 - 4x + 4 = (x - 2)(x - 2)$
   - $x^2 + 10x + 25 = (x + 5)(x + 5)$
   - $x^2 + 4x + 4 = (x + 2)(x + 2)$
Explore Together

Grouping
Once students have completed all of the problems, have the groups share their methods and solutions.

Problem 2
A polynomial expression is an expression written in the form \(a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0\), where \(a\) is any real number and \(n\) is a positive integer. Students are instructed to factor several polynomial expressions using these methods. Discuss the explanations for greatest common factor and factoring by grouping and work through the provided examples as a class.

Grouping
Have students work in groups to complete Problem 2.

Problem 1
Special Products
6. What do you notice about the factors of a perfect square trinomial?
   A perfect square trinomial has two factors, which are identical.

Problem 2
Factoring Polynomial Expressions
A polynomial expression is an equation that can be written in the form \(a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_2x^2 + a_1x + a_0\), where \(a\) is any real number and \(n\) is a positive integer. A linear expression is a first degree polynomial expression because the highest power is 1. A quadratic expression is a second degree polynomial expression because the highest power is 2.

1. Factor each polynomial expression.

   a. \(x^2 - 5x + 4\)
      \(x^2 - 5x + 4\)
      \((x - 4)(x - 1)\)

   b. \(x^4 - 5x^2 + 4\)
      \(x^4 - 5x^2 + 4\)
      \((x^2 - 4)(x^2 - 1)\)
      \((x - 2)(x + 2)(x - 1)(x + 1)\)

   c. \(x^4 - 8x^2 + 16\)
      \(x^4 - 8x^2 + 16\)
      \((x^2 - 4)(x^2 - 4)\)
      \((x - 2)(x + 2)(x - 2)(x + 2)\)

   d. \(x^4 + 10x^2 + 9\)
      \(x^4 + 10x^2 + 9\)
      \((x^2 + 9)(x^2 + 1)\)

The greatest common factor of a polynomial is the largest factor that is common to all terms of the polynomial. Solving some polynomial expressions requires factoring out the greatest common factor and then factoring the remaining factor.

For example, factor the polynomial expression \(2x^3 - 8x\).

\[2x^3 - 8x\]

Greatest common factor: \(2x\)

\[2x(x^2 - 4)\]

\[2x(x - 2)(x + 2)\]
Problem 2

Factoring Polynomial Expressions

2. Factor each polynomial expression completely.

a. \(4x^3 - 36x\)
   
   Greatest common factor: \(4x\)
   
   \(4x^3 - 36x\)
   
   \(4x(x^2 - 9)\)
   
   \(4x(x - 3)(x + 3)\)

b. \(4x^3 - 4x^2 - 24x\)
   
   Greatest common factor: \(4x\)
   
   \(4x^3 - 4x^2 - 24x\)
   
   \(4x(x^2 - x - 6)\)
   
   \(4x(x - 3)(x + 2)\)

c. \(3x^3 - 27x^2 - 30x\)
   
   Greatest common factor: \(3x\)
   
   \(3x^3 - 27x^2 - 30x\)
   
   \(3x(x^2 - 9x - 10)\)
   
   \(3x(x - 10)(x + 1)\)

Factoring by grouping is another method of factoring. To factor by grouping, create two groups of terms and factor out the greatest common factor of each group. Then factor out the greatest common factor of the groups.

For example, factor the polynomial expression \(x^3 + 3x^2 - 4x - 12\).

\[x^3 + 3x^2 - 4x - 12\]

\[x^2(x + 3) - 4(x + 3)\]

\[(x^2 - 4)(x + 3)\]

\[(x - 2)(x + 2)(x + 3)\]
Explore Together

Grouping
After providing sufficient time for students to complete Problem 2, bring the groups back together to share their methods and solutions.

Essential Ideas
- Differences of two squares are special polynomials written in the form \( a^2 - b^2 = (a + b)(a - b) \), where \( a \) and \( b \) are any real numbers.
- Perfect square trinomials are special polynomials written in the form \( a^2 \pm 2ab + b^2 = (a \pm b)^2 \)
  or \( a^2 - 2ab + b^2 = (a - b)^2 \), where \( a \) and \( b \) are any real numbers.
- A polynomial expression is an expression written in the form \( a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x + a_0 \), where \( a_0, a_1, \ldots, \) and \( a_n \) are any real numbers and \( n \) is a positive integer.
- In a polynomial expression, the (non-zero) term having the highest power determines the degree of the polynomial expression.
- The greatest common factor of a polynomial is the largest factor that is common to all terms of the polynomial.
- To factor by grouping, create two groups of terms and factor the greatest common factor of each group. Then factor the greatest common factor of the groups.

Problem 2

Factoring Polynomial Expressions

3. Factor each polynomial expression completely.

a. \( x^3 + 4x^2 - 9x - 36 \)
   \( x^3 + 4x^2 - 9x - 36 \)
   \( x^2(x + 4) - 9(x + 4) \)
   \( (x^2 - 9)(x + 4) \)
   \( (x - 3)(x + 3)(x + 4) \)

b. \( x^3 - 5x^2 + 3x - 15 \)
   \( x^3 - 5x^2 + 3x - 15 \)
   \( x^2(x - 5) + 3(x - 5) \)
   \( (x^2 + 3)(x - 5) \)

c. \( 2x^4 + 4x^3 - 2x^2 - 4x \)
   \( 2x^4 + 4x^3 - 2x^2 - 4x \)
   \( 2x(x^3 + 2x^2 - x - 2) \)
   \( 2x(x^2 - 1)(x + 2) \)
   \( 2x(x - 1)(x + 1)(x + 2) \)

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 10.6 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 10.6 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 10.

Check Students' Understanding
Match each polynomial expression with its simplified form.

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<th>Radical Expression</th>
<th>Simplification</th>
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<td>1.</td>
<td>(D) $3x^3 + 15x^2 - 72x$</td>
<td>A. $3x(x - 1)(x + 3)(x - 8)$</td>
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<tr>
<td>2.</td>
<td>(E) $3x^4 + 30x^3 + 39x^2 - 72x$</td>
<td>B. $(x - 1)(x + 3)(x + 8)$</td>
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<td>3.</td>
<td>(C) $x^4 + 11x^2 + 24$</td>
<td>C. $(x^2 + 8)(x^2 + 3)$</td>
</tr>
<tr>
<td>4.</td>
<td>(B) $x^3 + 10x^2 + 13x - 24$</td>
<td>D. $3x(x - 3)(x + 8)$</td>
</tr>
<tr>
<td>5.</td>
<td>(A) $3x^4 - 18x^3 - 57x^2 + 72x$</td>
<td>E. $3(x - 1)(x + 3)(x + 8)$</td>
</tr>
</tbody>
</table>
10.7 Fractions Again
Rational Expressions

Learning By Doing Lesson Map

Objectives
In this lesson, you will:
■ Simplify rational expressions.
■ Add, subtract, multiply, and divide rational expressions.

Key Terms
■ algebraic fraction
■ rational expression
■ excluded or restricted value
■ rational equation
■ extraneous solution

Sunshine State Standards
MA.912.A.2.13
Solve real-world problems involving relations and functions.
MA.912.A.4.3
Factor polynomial expressions.
MA.912.A.5.1
Simplify algebraic ratios.
MA.912.A.10.1
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.
MA.912.A.10.2
Decide whether a solution is reasonable in the context of the original situation.
MA.912.A.10.3
Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

Essential Ideas
■ An algebraic fraction or rational expression is a fraction that contains an algebraic expression in the numerator, the denominator, or both.
■ An excluded or restricted value is a value of the variable that results in a zero in the denominator of a rational expression.
■ To simplify a rational expression, factor the numerator and denominator completely. Then divide out factors common to the numerator and the denominator. State all excluded values of the original rational expression.
■ A rational equation is an algebraic equation with one or more rational expressions.
■ To solve a rational equation, determine the least common denominator for all the rational expressions, multiply each term by the least common denominator, and solve the resulting equation.
■ Multiplying a rational equation by an expression containing a variable may introduce solutions called extraneous solutions to the resulting equation that are not solutions to the original equation.

Essential Questions
1. What is a rational expression?
2. What is a restricted value?
3. What are the steps to simplify a radical expression?
4. What is a rational equation?
5. What are the steps to solve a rational equation?
6. What are extraneous solutions and how are they created?
Show The Way

Warm Up

Determine the least common denominator.

1. \( \frac{2}{x} + \frac{8}{9} \quad \text{LCD} = 9x \)

2. \( \frac{3}{x} + \frac{2}{3x} \quad \text{LCD} = 3x \)

3. \( \frac{2}{x+4} + \frac{3}{4} \quad \text{LCD} = 4(x+4) \)

4. \( \frac{2}{x+3} + \frac{8}{x-3} \quad \text{LCD} = (x+3)(x-3) \)

5. \( \frac{1}{x+5} + \frac{4}{x^2+5x} \quad \text{LCD} = x(x+5) \)

Motivator

If zero is in the denominator of a fraction, the fraction is undefined.

What value(s) of \( x \) would result in undefined solutions in each of the following equations?

- \( f(x) = \frac{2}{(x+4)} + \frac{3}{4} \quad x \neq -4 \)
  
  Using \( x = -4 \) would result in an undefined solution.

- \( f(x) = \frac{1}{(x-3)} + \frac{2}{5} \quad x \neq 3 \)
  
  Using \( x = 3 \) would result in an undefined solution.

- \( f(x) = \frac{5}{6x} - \frac{2}{7} \quad x \neq 0 \)
  
  Using \( x = 0 \) would result in an undefined solution.

- \( f(x) = \frac{9}{25 - x^2} - \frac{1}{5} \quad x \neq 5, -5 \)
  
  Using \( x = 5 \) or \( x = -5 \) would result in an undefined solution.

- \( f(x) = \frac{9}{x^2 - x - 42} + \frac{3}{4x + 8} \quad x \neq -6, -2, 7 \)
  
  Using \( x = -6, x = -2, \) or \( x = 7 \) would result in an undefined solution.
Explore Together

Problem 1

The definition of an algebraic fraction or rational expression is provided for students, with explanations of excluded or restricted values. Students are reminded that division by zero is undefined; therefore excluded variables are easily identified if attention is focused on the denominators of the rational expressions. Students are also given directions and an example for simplifying rational expressions.

Grouping

Have students work in groups to complete Problem 1.

After providing sufficient time for students to complete Problem 1, bring the groups back together to share their methods and solutions.

Problem 1

Rational Expressions

An algebraic fraction or a rational expression is a fraction that contains an algebraic expression in the numerator, the denominator, or both. An excluded or restricted value is a value of the variable that results in a zero in the denominator of a rational expression.

Remember that division by zero is undefined. For example, the rational expression \( \frac{4}{x} \) is undefined when \( x = 0 \).

1. Identify the excluded values for each rational expression.
   a. \( \frac{5x}{x - 4} \) \( x \neq 0 \)
   b. \( \frac{x + 2}{9x} \) \( x \neq -2 \)
   c. \( \frac{9x}{x - 4} \) \( x \neq 4 \)
   d. \( \frac{x^2 + 2x - 3}{x - 7} \) \( x \neq -3, 1 \)
   e. \( \frac{x^2 + 7x}{x^2 + 7x} \) \( x \neq -7, 0 \)

To simplify a rational expression, factor the numerator and denominator completely. Then divide out factors common to the numerator and the denominator. State all excluded values of the original rational expression. For example, \( \frac{4x}{x^2 - 1} = \frac{4 \cdot x}{(x - 1)(x + 1)} = \frac{4}{x - 1} \), \( x \neq 0, 1 \)

2. Simplify each rational expression completely. Be sure to include excluded values.
   a. \( \frac{10x}{15x} = \frac{1}{x} \cdot \frac{5 \cdot 2 \cdot x}{5 \cdot 3 \cdot x} = \frac{2}{3} \), \( x \neq 0 \)
   b. \( \frac{7x}{2x^2 - 2x} = \frac{7 \cdot x}{2 \cdot x \cdot (x - 1)} = \frac{7}{2(x - 1)} \), \( x \neq 0, 1 \)
   c. \( \frac{5x^2 - 5x}{x^2 - 2x + 1} = \frac{5x(x - 1)}{(x - 1)(x - 1)} = \frac{5x}{x - 1} \), \( x \neq 1 \)
   d. \( \frac{6x + 25}{x^2 + 5x} = \frac{5(x + 5)}{x(x + 5)} = \frac{5}{x} \), \( x \neq -5, 0 \)
   e. \( \frac{x^2 - 5x + 6}{x^2 + 3x - 18} = \frac{(x - 2)(x - 3)}{(x + 6)(x - 3)} = \frac{x - 2}{x + 6} \), \( x \neq -6, 3 \)
   f. \( \frac{5x^2 - 5x}{x^2 - 2x + 1} = \frac{5x(x - 1)}{(x - 1)(x - 1)} = \frac{5x}{x - 1} \), \( x \neq 1 \)
Explore Together

Problem 2

Students are given the definition of rational equations as algebraic equations with one or more rational expressions. An example of solving a rational equation is provided and students are instructed to first determine the least common denominator, considering all terms of the rational equation. They must then multiply each term of the equation by the LCD and attempt to solve the resulting equation.

Grouping

Have students work in groups to complete Problem 2.

Problem 2

Rational Equations

A rational equation is an algebraic equation with one or more rational expressions. To solve a rational equation:

- Determine the least common denominator, LCD, for all the rational expressions.
- Multiply each term by the LCD.
- Solve the resulting equation. Identify any excluded values.
- Check solution(s).

For example, solve \( \frac{4}{x} + \frac{5}{2x} = \frac{13}{12} \)

\[ \text{LCD} = 12x \]

\[ 12x \left( \frac{4}{x} \right) + 12x \left( \frac{5}{2x} \right) = 12x \left( \frac{13}{12} \right) \]

\[ x \neq 0 \]

\[ 48 + 30 = 13x \]

\[ 78 - 13x \]

\[ 6 = x \quad x \neq 0 \]

Check:

\[ \frac{4}{6} + \frac{5}{2} = \frac{8}{12} + \frac{5}{12} = \frac{13}{12} \]

1. Solve each rational equation. Check all solutions by substituting the values into the original equation.

a. \( \frac{5}{x} + \frac{5}{2} = -\frac{1}{4x} \)

\[ \text{LCD} = 4x \]

\[ 4x \left( \frac{5}{x} \right) + 4x \left( \frac{5}{2} \right) = 4x \left( -\frac{1}{4x} \right) \]

\[ 24 + 10x = -1 \]

\[ 10x = -25 \]

\[ x = -\frac{5}{2} \]

Check:

\[ \frac{5}{2} + \frac{5}{2} = -\frac{1}{4 \left( \frac{5}{2} \right)} \]

\[ \frac{12}{2} + \frac{10}{10} = \frac{24}{10} + \frac{25}{10} = \frac{49}{10} \]

b. \( \frac{2x}{x - 2} = \frac{10}{3} \)

\[ \text{LCD} = 3(x - 2) \]

\[ 3(x - 2) \left( \frac{2x}{x - 2} \right) = 3(x - 2) \left( \frac{10}{3} \right) \]

\[ x \neq 2 \]

\[ 6x = 10x - 20 \]

\[ -4x = -20 \]

\[ x = 5 \quad x \neq 2 \]

Check:

\[ \frac{2(5)}{5 - 2} = \frac{10}{3} \]
Explore Together

Problem 2

Rational Equations

c. \[
\frac{14}{x - 2} - \frac{12}{x + 3} \]
\[\text{LCD} = (x + 3)(x - 2)\]
\[\frac{14}{x - 2} = \frac{14}{x + 3} \cdot \frac{12}{x - 2} \quad x \neq -3, 2\]
\[14x + 42 = 12x - 24\]
\[2x = -66\]
\[x = -33 \quad x \neq -3, 2\]
Check:
\[
\frac{14}{-33 - 2} = \frac{14}{-33 + 3}
\]
\[
\frac{14}{-35} = \frac{14}{-30}
\]
\[
\frac{2}{5} = \frac{2}{5}
\]
d. \[
\frac{x}{x - 4} - \frac{x}{x + 4}
\]
\[\text{LCD} = (x + 4)(x - 4)\]
\[\frac{x}{x - 4} - \frac{x}{x + 4} = \frac{x}{x + 4} \cdot \frac{1}{x - 4} \quad x \neq -4, 4\]
\[x^2 + 4x = x^2 - 4x\]
\[8x = 0\]
\[x = 0 \quad x \neq -4, 4\]
Check:
\[
\frac{0}{0 - 4} = \frac{0}{0 + 4}
\]
\[
0 = 0
\]
e. \[
\frac{x}{x + 3} = \frac{8}{x + 6}
\]
\[\text{LCD} = (x + 3)(x + 6)\]
\[\frac{x}{x + 3} = \frac{8}{x + 6} \cdot \frac{x}{x + 3} \quad x \neq -6, -3\]
\[x^2 + 6x = 8x + 24\]
\[x^2 - 2x - 24 = 0\]
\[x = -4 \quad \text{or} \quad x = 6 \quad x \neq -6, -3\]
Check:
\[
\frac{-4}{-4 + 3} = \frac{8}{-4 + 6} \quad \text{or} \quad \frac{6}{6 + 3} = \frac{8}{6 + 6}
\]
\[
4 = 4 \quad \text{or} \quad 2 \cdot 3 = 3
\]
Explore Together

Note Extraneous solutions are explained as solutions that result from multiplying a rational equation by an expression containing a variable and that are not to be considered as actual solutions to the original equation, but rather as solutions to the secondary equation formed by the multiplication.

Problem 2 Rational Equations

**f.** \( \frac{x}{x - 1} - \frac{2}{x} = \frac{1}{x - 1} \)

\[ \text{LCD} = x(x - 1) \]

\[ x(x - 1) \left( \frac{x}{x - 1} \right) - x(x - 1) \left( \frac{2}{x} \right) = \]

\[ x(x - 1) \left( \frac{1}{x - 1} \right) \quad x \neq 0, 1 \]

\[ x^2 - 2x + 2 = x \]

\[ x^2 - 3x + 2 = 0 \]

\[ (x - 2)(x - 1) = 0 \]

\[ x = 2 \quad \text{or} \quad x = 1 \quad x \neq 0, 1 \]

**Check:**

\[ \frac{2}{2 - 1} - \frac{2}{2 - 1} = \frac{1}{2 - 1} - \frac{1 - 2}{1 - 1} \]

\[ 1 - 1 \quad \text{or} \quad \text{undefined division by zero} \]

2. How is the problem in Question 1(f) different from the others in Question 1?

The equation appears to have two solutions but one solution does not check.

Multiplying a rational equation by an expression containing a variable may introduce solutions, called **extraneous solutions**, to the resulting equation that are not solutions to the original equation. Always check all solutions for extraneous solutions.

3. Solve each rational equation.

a. \( \frac{6}{x - 2} = \frac{12}{x^2 - 2x} - \frac{3}{x} \)

\[ \text{LCD} = x(x - 2) \quad x \neq 0, 2 \]

\[ x(x - 2) \left( \frac{6}{x - 2} \right) = x(x - 2) \left( \frac{12}{x^2 - 2x} \right) - x(x - 2) \left( \frac{3}{x} \right) \]

\[ 6x = 12 - 3x + 6 \]

\[ 9x = 18 \]

\[ x = 2 \]

2 is an extraneous root so no solution
Grouping
After providing sufficient time for students to complete Problem 2, bring the groups back together to share their methods and solutions.

Essential Ideas
- An algebraic fraction or rational expression is a fraction that contains an algebraic expression in the numerator, the denominator, or both.
- An excluded or restricted value is a value of the variable that results in a zero in the denominator of a rational expression.
- To simplify a rational expression, factor the numerator and denominator completely. Then divide out factors common to the numerator and the denominator. State all excluded values of the original rational expression.
- A rational equation is an algebraic equation with one or more rational expressions.
- To solve a rational equation, determine the least common denominator for all the rational expressions, multiply each term by the least common denominator, and solve the resulting equation.
- Multiplying a rational equation by an expression containing a variable may introduce solutions called extraneous solutions to the resulting equation that are not solutions to the original equation.

Problem 2
Rational Equations

b. \( \frac{x - 2}{x - 2} = \frac{-2 - x}{x - 2} \)

\[
\text{LCD} = x - 2 \quad x \neq 2 \\
(x - 2)(x - 2) \left( \frac{2}{x - 2} \right) = -(x - 2)(2) - (x - 2) \left( \frac{x}{x - 2} \right) \\
x^2 - 2x - 2 = -2x + 4 - x \\
x^2 + x - 6 = 0 \\
(x - 2)(x + 3) = 0 \\
x = -3, 2 \\
\]

Check:

\[
\begin{align*}
(-3) - 2 &= -2 - \frac{(-3)}{2} \\
-3 + \frac{2}{3} &= -2 - \frac{3}{5} \\
\frac{-2}{3} &= \frac{-2}{3} \\
\frac{2}{(2) - 2} &= \frac{-2}{(2) - 2} \\
2 \text{ is an extraneous root} \\
\text{Solution: } x &= -3 \\
\end{align*}
\]

Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 10.7 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 10.7 in the Student Assignments book for additional resources. See the Teacher's Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher's Resources and Assessments book for Chapter 10.7.

Check Students' Understanding
Match rational equation with the correct solution(s) and extraneous root(s).

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<th></th>
<th>Rational Equation</th>
<th>Solutions and Extraneous Roots</th>
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<tr>
<td>1. (E)</td>
<td>[ \frac{6}{x-3} = \frac{18}{x^2-3x} - \frac{3}{x} ]</td>
<td>A. Solution ( x = -4 ) Extraneous root ( x = 3 )</td>
</tr>
<tr>
<td>2. (A)</td>
<td>[ x - \frac{3}{x-3} = -3 - \frac{x}{x-3} ]</td>
<td>B. Solution ( x = 1 ) ( x \neq 0 ) Extraneous root ( x = 3 )</td>
</tr>
<tr>
<td>3. (B)</td>
<td>[ \frac{3}{x-3} - \frac{3}{x} = \frac{1}{x-3} + \frac{6}{x(x-3)} ]</td>
<td>C. Solution ( x = 12 ) ( x \neq 0, 3 )</td>
</tr>
<tr>
<td>4. (C)</td>
<td>[ \frac{3}{x-3} = \frac{4}{x} ]</td>
<td>D. Solution ( x = 12 ) ( x \neq 0 )</td>
</tr>
<tr>
<td>5. (D)</td>
<td>[ \frac{3}{x} + \frac{1}{3} = \frac{42}{6x} ]</td>
<td>E. No Solution Extraneous root ( x = 3 )</td>
</tr>
</tbody>
</table>
Objectives
In this lesson, you will:
■ Divide polynomials using long division.
■ Divide polynomials using synthetic division.

Key Terms
■ Fundamental Theorem of Algebra
■ synthetic division

Sunshine State Standards
MA.912.A.4.4
Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

MA.912.A.10.1
Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing-and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

MA.912.A.10.2
Decide whether a solution is reasonable in the context of the original situation.

Essential Ideas
■ The Fundamental Theorem of Algebra states a polynomial function of degree \( n \) has \( n \) zeros. Every polynomial of degree \( n \) can be written as the product of \( n \) factors of the form \( ax + b \).
■ Long division can be used to divide two polynomials and is similar to long division as used to divide two numbers.
■ Factors of polynomials must divide into the polynomial evenly with a remainder of zero.
■ If the remainder when dividing two polynomials is zero, then the divisor divides into the dividend evenly and the dividend can be written as the product of the divisor and the quotient.
■ If the remainder when dividing two polynomials is not zero, then the divisor does not divide into the dividend evenly and the dividend can be written as the product of the divisor and the quotient plus the remainder.
■ When dividing polynomials, any missing terms must be added with 0 as a coefficient to keep the powers aligned.
■ Synthetic division is an alternate method for dividing polynomials but can be used only if the dividend is a linear factor of the form \( x - r \).

Essential Questions
1. What is the Fundamental Theorem of Algebra?
2. What method is used to divide two polynomials?
3. What is the remainder when factors of polynomials are divided into the polynomial?
4. What is true about the relationship between the divisor, quotient, and dividend if the remainder is zero after dividing the two polynomials?
5. If the remainder is not zero, after dividing two polynomials, how is the answer expressed?
6. When dividing two polynomials, what needs to be done if some terms are missing?
7. Can synthetic division be used anytime?
Show The Way

Warm Up
Perform the necessary long division and write each dividend as a product of the quotient and divisor plus any remainder.

1. $\frac{28}{8} \div 224 = (8)(28)$
   
   
   
   
   
   

2. $\frac{45}{31} \div 1395 = (31)(45)$
   
   
   
   
   

3. $\frac{13}{22} \div 289 = (22)(13) + 3$
   
   
   
   
   

Motivator
Perform long division to solve each of the following.

$\frac{185}{5}\div 925$

$\frac{12}{14} \div 231$

$\frac{16}{14} \div 228$

$\frac{24}{22}$

$\frac{48}{48}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$

$\frac{0}{20}$
Explore Together

Problem 1

Begin this problem as a class discussion. First introduce the Fundamental Theorem of Algebra, which states that a polynomial function of degree \(n\) has \(n\) zeroes. So, every polynomial of degree \(n\) can be written as the product of \(n\) factors of the form \(ax + b\). Remind students that a factor of a number must divide into the number evenly with a remainder of zero and polynomial factors share the same property. As a class, go over the two examples provided. One example is numeric long division and the other is polynomial long division. The two examples are positioned side by side for ease of comparison. Students should note that the process remains essentially unchanged whether they are working with numbers or polynomials.

Problem 1  Dividing Polynomials

The Fundamental Theorem of Algebra states that a polynomial function of degree \(n\) has \(n\) zeroes. So, every polynomial of degree \(n\) can be written as the product of \(n\) factors of the form \(ax + b\). For example, 

\[ f(x) = 2x^2 - 3x - 9 = (2x + 3)(x - 3). \]

A factor of a number must divide into the number evenly with a remainder of zero. Factors of polynomials have the same property. Long division can be used to divide two polynomials, similar to using long division to divide two numbers. For example:

\[
\begin{align*}
\frac{-3}{2x^2 - 3x - 9} & \quad \text{(Divide: } x + \frac{2}{3}) \quad \frac{3}{25}\frac{775}{775} \quad \text{(Divide: } 2)\frac{7}{7}\n\frac{-3}{2x - 6} & \quad \text{(Multiply: } 2x(x - 3)) \quad \frac{3}{25}\frac{775}{775} \quad \text{(Multiply: } 3(25)) \quad \frac{75}{75} \quad \frac{2}{2}
\frac{-2}{3x} & \quad \text{(Subtract)} \quad \frac{75}{75} \quad \frac{2}{2}
\frac{-2}{x} & \quad \text{(Bring down)} \quad \frac{75}{75} \quad \frac{25}{25}
\frac{-3}{2x^2 - 3x - 9} & \quad \text{(Divide: } x + \frac{3}{3}) \quad \frac{3}{25}\frac{775}{775} \quad \text{(Divide: } 2)\frac{1}{1}\n\frac{-3}{2x - 6} & \quad \text{(Multiply: } 3(x - 3)) \quad \frac{3}{25}\frac{775}{775} \quad \text{(Multiply: } 3(25)) \quad \frac{75}{75} \quad \frac{25}{25}
\frac{3}{3x - 9} & \quad \frac{75}{75} \quad \frac{25}{25}
\frac{2x + 3}{3x - 9} & \quad \frac{31}{31}
\frac{3}{3x - 9} & \quad \frac{25}{25}
\frac{3}{3x - 9} & \quad \frac{25}{25}
\frac{0}{0}
\end{align*}
\]
Explore Together

Grouping
Following the class discussion, have students work in groups to complete Problem 1.

Problem 1  Dividing Polynomials

1. Calculate each quotient.
   a. $\frac{x^4 + 6x + 19}{x^3 - 4x^2}$
      $\frac{6x^2 - 5x}{6x^2 - 24x}$
      $\frac{19x + 6}{19x - 76}$
      82
   b. $\frac{x^3 + x + 1}{x^2 - x^2}$
      $\frac{x^3 + 0x}{x^2 - x}$
      $\frac{x - 1}{x - 1}$
      0
      $2x^2 + 3x^2 + 7x + 7$
   c. $\frac{2x - 3}{4x^4 - 6x^3}$
      $\frac{6x^2 + 5x^2}{6x^2 - 9x^2}$
      $\frac{14x^2 - 7x}{14x^2 - 21x}$
      $\frac{14x + 9}{14x - 21}$
      30
      $3x^2 - x^2 + 2x + 1$
   d. $\frac{3x + 2}{9x^4 + 6x^2}$
      $\frac{-3x^3 + 4x^2}{-3x^3 - 2x^2}$
      $\frac{6x^2 + 7x}{6x^2 + 4x}$
      $\frac{3x + 2}{3x + 2}$
      0

2. What do you notice about the dividends in Questions 1(b) and 1(c)? Why is this important?
   Missing terms with 0 coefficients were added. This is necessary to keep the powers aligned.

3. Is each divisor in Question 1 a factor of the dividend? Explain.
   Not every divisor is a factor of the dividend. If the remainder is not zero, then the divisor is not a factor.
Note: Some problems emphasize that the dividend can be expressed as the product of the quotient and the divisor plus any remainder.

Explore Together

4. What is the remainder when you divide a polynomial by one of its factors?
   The remainder is 0.

   If the remainder when dividing two polynomials is zero, then the divisor divides into the dividend evenly and the dividend can be written as the product of the divisor and the quotient.

   If the remainder when dividing two polynomials is not zero, then the divisor does not divide into the dividend evenly and the dividend can be written as the product of the divisor and the quotient plus the remainder.

5. For each problem in Question 1 that did divide evenly, write the dividend as the product of the divisor and the quotient.
   \[ x^2 + 2x - 1 = (x - 1)(x + 1) \]
   \[ 9x^4 + 3x^2 + 4x + 7x + 2 = (3x + 2)(3x^2 - x^2 + 2x + 1) \]

6. For each problem in Question 1 that did not divide evenly, write the dividend as the product of the divisor and the quotient plus the remainder.
   \[ x^3 + 2x^2 - 5x + 6 = (x - 4)(x^2 + 6x + 19) + \frac{32}{x - 4} \]
   \[ 4x^4 + 2x^2 - 7x + 9 = (2x - 3)(2x^3 + 3x^2 + 7x + 7) + \frac{30}{2x - 3} \]

7. Perform each division. Then write the dividend as the product of the divisor and the quotient plus the remainder.
   a. \[ \frac{4x^2 + 5x^2 + 15x + 32}{x - 2} \]
      \[ \frac{4x^4 - 8x^3 + 5x^2 - 10x^2}{4x^4 - 8x^3} \]
      \[ \frac{5x^2 - 2x}{5x^2 - 2x} \]
      \[ \frac{15x^2 + 30x}{15x^2 + 30x} \]
      \[ \frac{15x^2 - 1}{15x^2 - 1} \]
      \[ \frac{32x - 64}{32x - 64} \]
      \[ \frac{63}{63} \]
      \[ 4x^4 - 3x^3 + 5x^2 + 2x - 1 = (x - 2)(4x^3 + 5x^2 + 15x + 32) + \frac{63}{x - 2} \]
Explore Together

Grouping
After providing sufficient time for students to complete Problem 1, bring the groups back together to share their methods and solutions.

Begin this problem as a class discussion. First introduce synthetic division as an alternate method for dividing polynomials. Emphasize that synthetic division can only be performed if the dividend is a linear factor of the form \( x - r \). As a class, review the example of synthetic division provided.

Problem 1: Dividing Polynomials

- \( 3x^3 - 11x^2 + 33x - 97 \)
- \( x + 3 \)
- \( 3x^2 - 2x^2 + 9x + 2x - 1 \)

\[
\frac{3x^4 + 9x^3}{-11x^3 + 9x^2} \quad \frac{3x^3 + 2x}{-11x^2 + 33x^2} \quad \frac{33x^2 + 99x}{-97x - 1} \quad \frac{-97x - 291}{290}
\]

\[
x^2 - 2x^2 + 9x + 2x - 1 = \frac{290}{x + 3} \left( (3x^3 - 11x^2 + 33x - 97) + \frac{290}{x + 3} \right)
\]

Problem 2: Synthetic Division

In 1809, Paolo Ruffini introduced a method for dividing two polynomials called synthetic division. Synthetic division can be used only if the divisor is a linear factor of the form \( x - r \). The following example compares long division and synthetic division.

\[
\begin{array}{c|ccc}
3 & 2 & -3 & -9 \\
\hline
2 & 2 & -3 & -9 \\
\hline
1 & 6 & 3 & -9 \\
\hline
2 & -3 & -9 \\
\hline
2 & -3 & -9 \\
\hline
3 & 6 & 9 \\
\hline
2 & 3 & 0
\end{array}
\]

Bring down the 2
Multiply 3 by 2 and place in the next column
Add the values in the second column
Repeat the process until complete
The quotient is \( 2x + 3 \) and the remainder is 0.

As with long division, every power must have a place holder.
Explore Together

Grouping
Have students work in groups to complete Problem 2.

Note Remind students that every power must have a place holder. Students generally prefer synthetic division over long division because it is faster and cumbersome variables are not written in each step.

Problem 2 Synthetic Division

1. For each synthetic division, write the dividend as the product of the divisor and the quotient plus the remainder.
   
a. \[2 \begin{array}{ccc}
   1 & 0 & -3 & 6 \\
   2 & 4 & 0 & -6 \\
   1 & 2 & 0 & -3 & 0 \\
   \end{array}\]
   \[x^4 + 2x^2 - 4x^2 - 3x + 6 = (x - 2)(x^3 + 2x^2 + 0x - 3)\]

b. \[-3 \begin{array}{ccc}
   2 & -4 & -4 & -3 & 6 \\
   -6 & 30 & -78 & 243 \\
   2 & -10 & 26 & -81 & 249 \\
   \end{array}\]
   \[2x^4 - 4x^2 - 4x^2 - 3x + 6 = (x + 3)(2x^3 - 10x^2 + 26x - 81) + \frac{249}{x + 3}\]

2. Use synthetic division to calculate each quotient. Then write the dividend as the product of the divisor and the quotient plus the remainder.
   
a. \[x - 4 \begin{array}{ccc}
   1 & 2 & -5 & 6 \\
   4 & 4 & 24 & 76 \\
   1 & 6 & 19 & 82 \\
   \end{array}\]
   \[x^3 + 2x^2 - 5x + 6 = (x - 4)(x^2 + 6x + 19) + \frac{82}{x - 4}\]

b. \[f(x) - 3x^3 + 4x^2 + 8\] Divided by \(x + 2\)
   \[-2 \begin{array}{ccc}
   3 & 4 & 0 & 8 \\
   -6 & 4 & -8 \\
   3 & -2 & 4 & 0 \\
   \end{array}\]
   \[3x^2 + 4x^2 + 8 = (x + 2)(3x^2 - 2x + 4)\]

c. \[2x^2 - 5x^4 + 2x^2 - 6x + 7\] Divided by \(2x - 3\)
   \[\begin{array}{ccc}
   2 & -5 & 0 & 2 & -6 & 7 \\
   3 & -3 & -9 & 15 & 117 \\
   2 & -2 & -3 & 5 & -39 & -61 \\
   2x^2 - 5x^4 + 2x^2 - 6x + 7 = \left(x - \frac{3}{2}\right)x - \frac{61}{8}\]
   \[\left(2x^2 - 2x^3 - 2x^2 + \frac{5}{2}x - \frac{39}{4}ight)\frac{-8}{x - \frac{3}{2}}\]
Explore Together

Grouping

After providing sufficient time for students to complete Problem 2, bring the groups back together to share their methods and solutions.

Essential Ideas

- The Fundamental Theorem of Algebra states a polynomial function of degree $n$ has $n$ zeros. Every polynomial of degree $n$ can be written as the product of $n$ factors of the form $ax + b$.
- Long division can be used to divide two polynomials and is similar to long division as used to divide two numbers.
- Factors of polynomials must divide into the polynomial evenly with a remainder of zero.
- If the remainder when dividing two polynomials is zero, then the divisor divides into the dividend evenly and the dividend can be written as the product of the divisor and the quotient.
- If the remainder when dividing two polynomials is not zero, then the divisor does not divide into the dividend evenly and the dividend can be written as the product of the divisor and the quotient plus the remainder.
- When dividing polynomials, any missing terms must be added with 0 as a coefficient to keep the powers aligned.
- Synthetic division is an alternate method for dividing polynomials but can be used only if the dividend is a linear factor of the form $x - r$.

Problem 2

Synthetic Division

d. $g(x) = 4x^3 - 2x^2 + 4x - 2 + 11$
   $r(x) = 2x + 1$

   Calculate $\frac{g(x)}{r(x)}$.

   $\begin{array}{c|cccc}
       -1 & 4 & 0 & -2 & 4 & -2 & 11 \\
       \hline
       & 4 & -2 & 1 & 9 & 17 \\
       -2 & 4 & -2 & -1 & 9 & 17 & 105 \\
   \end{array}$

   $g(x) = \left(4x^4 - 2x^3 + 9x^2 - 17\right) + \frac{105}{x + 1}$

   Be prepared to share your solutions and methods.
Follow Up

Assignment
Use the Assignment for Lesson 10.8 in the Student Assignments book. See the Teacher’s Resources and Assessments book for answers.

Skills Practice
Refer to the Skills Practice worksheet for Lesson 10.8 in the Student Assignments book for additional resources. See the Teacher’s Resources and Assessments book for answers.

Assessment
See the Assessments provided in the Teacher’s Resources and Assessments book for Chapter 10.

Check Students’ Understanding
Use synthetic division or long division to calculate each quotient. Match each division problem with the correct quotient.

<table>
<thead>
<tr>
<th>Division Problem</th>
<th>Quotient</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (E) $3x + 5 \overline{</td>
<td>3x^2 - x - 10}$</td>
</tr>
<tr>
<td>2. (C) $x + 2 \overline{</td>
<td>3x^3 + 11x + 10}$</td>
</tr>
<tr>
<td>3. (A) $x + 2 \overline{</td>
<td>3x^3 + 4x^2 - 4x}$</td>
</tr>
<tr>
<td>4. (B) $3x + 5 \overline{</td>
<td>3x^3 + 11x^2 + 10x}$</td>
</tr>
<tr>
<td>5. (D) $x + 2 \overline{</td>
<td>3x^3 + 8x^2 + 4x}$</td>
</tr>
</tbody>
</table>